Antitrust Limits to Patent Settlements: Appendix

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Output Game Between Patent Holder and Challenger

If the output levels are x_1 and x_2, and if the maximum profits that the incumbent can earn facing output level x_2 by the entrant are given by $\pi_1^*(x_2) = \max_{x_1} \pi_1(x_1, x_2)$, then the infringement damages owed to the patent holder (if infringement is found) are given by $\pi_M - \pi_1^*(x_2)$. Note that this damages rule has the attractive property that it gives the incumbent the incentive to set output to maximize profits, ignoring damages, which are independent of x_1.[1] So, the incumbent operates using its normal Cournot best-response function.

However, the entrant's behavior is definitely influenced by the prospect of infringement damages (so long as $\theta > 0$). Given x_1, the entrant's expected profits are given by

$$V_2(x_2) = \pi_2(x_1, x_2) - \theta(\pi_M - \pi_1^*(x_2)).$$

[1] With Cournot competition, the patent holder's best-response function is the same if the damages rule specifies actual damages rather than just damages that could not have been avoided given the conduct of the infringing firm. Formally, if actual damages are awarded, given x_2, the patent holder maximizes $\pi_1(x_1, x_2) + \theta(\pi_M - \pi_1(x_1, x_2))$, which is the same as just maximizing $\pi_1(x_1, x_2)$ as the patent holder does under the stronger mitigation rules.

Using the envelope theorem, we know that $\frac{d\pi_1^*(x_2)}{dx_2} = \frac{\partial \pi_1(x_1^*, x_2)}{\partial x_2}$, so the first-order equation for x_2 is given by $\frac{\partial \pi_2(x_1^*, x_2)}{\partial x_2} + \theta \frac{\partial \pi_1(x_1^*, x_2)}{\partial x_2} = 0$. The usual equation for the incumbent's optimal output applies: $\frac{\partial \pi_1(x_1, x_2^*)}{\partial x_1} = 0$.

If we have linear demand, $D(p) = A - p$ and constant marginal production costs of c_1 and c_2, then the firms' output in the resulting Cournot equilibrium are given by $x_1^* = \frac{A - 2c_1 + c_2}{3-\theta}$ and $x_2^* = \frac{A - 2c_2 + c_1 - \theta(A - c_1)}{3-\theta}$, with total output of

$$x^* = \frac{2A - c_1 - c_2 - \theta(A - c_1)}{3-\theta}$$

Naturally, when the patent is very weak, so $\theta \approx 0$, we get back the standard Cournot equilibrium. When the patent is strong, so $\theta \approx 1$, the challenger only produces if it is more efficient that the patent holder $(c_2 < c_1)$. If the challenger is not more efficient, this equation gives us back the monopoly output level of the patent holder; if the challenger is more efficient, we get the monopoly output level of the challenger. Effectively, the challenger maximizes profits for its lower level of costs and then compensates the patent holder for its own (lower) level of monopoly profits. This is one of many cases in which infringement by a more efficient firm is optimal so long as damages are equal to lost profits, not a multiple of lost profits (as in fact can occur for willful infringement).

Focusing now on the case in which the two firms are equally efficient, so $c_1 = c_2 = c$, the interim output level is given by $x^* = \frac{2-\theta}{3-\theta}(A - c)$. As θ ranges from zero to one, output ranges from $\frac{2}{3}(A - c)$, the

Cournot output level, down to $\frac{1}{2}(A-c)$, the monopoly output level. Since consumer surplus as a function of output is $x^2/2$, consumer surplus generated by competition in the shadow of liability varies with patent strength according to $S_L(\theta)=\frac{(A-c)^2}{2}\left(\frac{2-\theta}{3-\theta}\right)^2$.

For any level of patent strength, expected consumer surplus is higher under interim competition than it will be (on average) following the resolution of the patent dispute. Expected consumer surplus after resolution of the dispute equals $(A-c)^2(\frac{\theta}{8}+\frac{2(1-\theta)}{9})$, since surplus under monopoly is $(A-c)^2/8$ and surplus under Cournot competition is $2(A-c)^2/9$. Comparing these functions, we find for all values of θ between zero and one, consumer surplus is higher under interim competition than it will be on average after the resolution of the patent dispute.

The Patent Competition Index during the period when the firms are competing in the shadow of possible liability, PCI_L, is not difficult to compute in this case: $PCI_L=\frac{9}{7}(4(\frac{2-\theta}{3-\theta})^2-1)$. Of course, the index varies from one, when $\theta=0$, down to zero, when $\theta=1$. But note that PCI_L is concave, not linear, in θ, and thus exceeds the probability of non-infringement, $1-\theta$, for all interior values of θ.