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Econ 160: Midterm Exam

Winter Quarter, 2003

DIRECTIONS (Read Carefully!!):

- This exam includes **4 questions**. Please answer each question in a **separate blue book**. I have tried to order the questions according to their difficulty. It is strongly recommended to read the whole exam before you attempt to solve it.
- Please hand in your answers in a comprehensible format; **illegible answers may lose valuable points**! Also any answer that is not supported by calculations or justifications (when relevant) will receive reduced credit!
- Starting time: **1:15 PM** Ending time: **3:05 PM**

GOOD LUCK!!

Question 1: Basic Concepts (20 points)

Define, using as best you can the notation introduced in class, and briefly explain the following terms (4 points each):

(a) Best response

(b) Nash Equilibrium

For each of the following statements, provide a proof if it is true or a counter-example if it is not (4 points each):

(c) In a two-player matrix game, the process of iterated elimination of strictly dominated strategies will *always lead* to a pure-strategy Nash equilibrium

(d) In a game of perfect information there is *always* a unique subgame perfect equilibrium.

(e) Every *Strict Dominant Strategy Equilibrium* is a Nash equilibrium.

Question 2: Brand Location (25 points)

A potential entrant (firm 1) is considering entering a market that currently has a single incumbent (firm 2). If it plays "Out" (O), then the incumbent remains a monopolist and gets a payoff of 2, while the potential entrant gets a payoff of 0. If, however, the potential entrant enters by playing "In" (I), then both firms must simultaneously choose if they will produce for the "Low end" of the market (L) or the "High end" (H).

Both firms suffer a loss if they choose the same product: their payoffs are -6 each if they both choose the low end, and -3 if they both choose the high end. If they choose different products then the firm who chose the high end gets a payoff of 1, while the other gets -1.

(a) Draw the game-tree that represents the extensive form of this game, and identify the proper subgames. (5 points)

(b) Write down the matrix form of this game. (5 points)

(c) Find all the pure Strategy subgame-perfect equilibria. (10 points)

(d) Is there a subgame-perfect equilibrium that includes mixed strategies in the post-entry subgame? If yes, provide it, if not, prove your claim. (5 points)

Question 3: Agenda Setting (25 points)

An agenda-setting game is described as follows. The "issue space" (set of possible policies) is an interval $X = [0, 5]$. An Agenda Setter (player 1) proposes an alternative $x \in X$ against the status quo $q = 4$. After player 1 proposes x, the Legislator (player 2) observes the proposal selects between the proposal x and the status quo q.

Player 1's most preferred policy is $a = 1$, and for any final policy $y \in X$, his payoff is given by

$$u_1(y) = 10 - |y - a|,$$

where $|y - a|$ denotes the absolute value of $(y - a)$. Player 2's most preferred policy is $l = 3$, and for any final policy $y \in X$, her payoff is given by

$$u_2(y) = 10 - |y - l|.$$

That is, each player prefers policies that are closer to their most preferred policy.

(a) Is this a game of perfect or imperfect information? (3 points)

(b) Write the game down as a normal form game. (5 points)

(c) Find a Subgame Perfect equilibrium of this game. Is it unique? (5 points)

(d) Find a Nash equilibrium that is not subgame perfect. (5 points)

(e) Is the Nash equilibrium you found in **(d)** unique? If yes, explain. If not, show *all the Nash equilibria* of this game. (7 points)

Question 4: Wasteful Shipping Costs? (30 points)

Consider two countries, A and B, each with a monopolist that owns the only coal mine in the country, and it produces coal. Let firm 1 be the one located in country A, and firm 2 the one in country B. Let q_i^j, $i \in \{1,2\}$ and $j \in \{A,B\}$ denote the quantity that firm i sells in country j. Consequently, let $q_i = q_i^A + q_i^B$ be the total quantity produced by firm $i \in \{1,2\}$, and let $q^j = q_1^j + q_2^j$ be the total quantity sold in country $j \in \{A,B\}$. The demand for coal in countries A and B is given respectively by,

$$p^j = 90 - q^j,\ j \in \{A,B\},$$

and the costs of production for each firm is given by,

$$c_i(q_i) = 10q_i\,,\ i \in \{1,2\}.$$

(a) Assume that the countries do not have a trade agreement and, in fact, imports in both countries are prohibited. This implies that $q_2^A = q_1^B = 0$ is set as a political constraint. What quantities q_1^A and q_2^B will both firms produce? (6 points)

Now assume that the two countries sign a free-trade agreement that allows foreign firms to sell in their countries without any tariffs. There are, however shipping costs. If firm i sells quantity q_i^j in the foreign country (i.e., firm 1 selling in B or firm 2 selling in A) then shipping costs are equal to $10q_i^j$. Assume further that *each firm* chooses a pair of quantities q_i^A, q_i^B simultaneously, $i \in \{1,2\}$, so that a profile of actions consists of four quantity choices.

(b) Model this as a normal form game. (8 points)

(c) Find a Nash equilibrium of the game you described in **(b)**. Is it unique? (8 points)

Now assume that before the game you described in **(b)** is played, the research department of firm 1 discovered that shipping coal with the current ships causes the release of pollutants. If the firm would disclose this report to the World-Trade-Organization (WTO) then the WTO would prohibit the use of the current ships. Instead, a new shipping technology would be offered that would increase shipping costs to $40q_i^j$ (instead of $10q_i^j$ as above).

(d) Would firm 1 be willing to release the information to the WTO? Justify your answer with an equilibrium analysis. (8 points)