Winter 2001 Prof. Steve Tadelis

# Econ 160 - Final Exam

**DIRECTIONS (Read Carefully!!)**:

This exam includes 4 questions (the points of each part are clearly marked on the left margin). **It is strongly recommended to read the whole exam before you attempt to solve it** since questions may not necessarily appear in order of difficulty. Answer each question in a **separate blue-book** (you can uses parts of the blue-books as scratch-pads, but be sure to differentiate your final answer from your preliminary scribbles!). Please hand in your answers in a comprehensible format; **illegible or <u>unsupported</u> answers will lose valuable points**!

**Date:** Monday, 3/18/01 **Starting time:** 8:30 AM **Ending time:** 11:30 AM

**GOOD LUCK!!**

## Question 1: Basic Concepts (22 points)

(a) Define, using formal notation (when possible), and **briefly** explain the following terms:

4 (i) Subgame perfect Equilibrium
6 (ii) Perfect Bayesian Equilibrium

(b) For each of the following statements, provide a proof if it is true or a counter-example if it is not:

4 (i) In any 2-player game of *incomplete information*, the only proper Subgame is the whole game.
4 (ii) In any game of *perfect information*, every Nash Equilibrium is also a Subgame Perfect equilibrium.
4 (iii) For any Bayesian game, every Subgame Perfect equilibrium is also a Bayesian Nash Equilibrium.

**Question 2: Joint Ventures** (28 points)

Two firms sign a joint venture agreement in which they will **split the revenue** from their project. Each firm must simultaneously choose an investment level, but they cannot jointly commit to their investment levels. That is, firm 1 chooses either High (H), Low (L) or No (N) investment, and firm 2 independently chooses its investment level to be one of {H, L, N} at the same time, and they cannot coordinate their efforts. The **revenue** of the project is 18 if both choose H, it is 12 if one firm chooses H and the other L, it is 6 if both choose L, and it is 0 if either firm chooses N. The cost for a firm is 5 if it chooses H, 1 if it chooses L, and 0 if it chooses N.

Assume first that this interaction is a one-shot project.

4 (a) Write this interaction as a normal (strategic) form game.

6 (b) Solve for the pure strategy Nash equilibria of this game.

2 (c) If the firm's can commit to investment levels different from the Nash Equilibrium, what would these investment levels be?

Now assume that these two firms have two such joint ventures sequentially (i.e., the above is repeated twice) where the outcomes of the first project are determined and observed by all before the second project's investments are chosen. The total payoffs are the discounted sum of payoffs of the two periods where the discount factor is $0 < \delta \leq 1$.

10 (d) For which values of $\delta$ can the investment levels you found in (c) above be supported as part of a pure strategy Subgame Perfect Equilibrium (SPE)? Precisely define the SPE you are using to support the outcome, and show that it is a SPE.

6 (e) Are there other pure-strategy SPE for the values of $\delta$ you found in (d) above? If no, prove it, if yes, give an example. Although this game is not an infinitely repeated game, can you relate your answer to this part to the folk theorem in repeated games?

**Question 3: Political Races** (26 points)

There are two politicians, an incumbent (1) and a potential rival (2) that are running for the local mayoral race. The incumbent has either a broad base of support (B) or a small base of support (S), each occurring with probability ½.. The incumbent *knows* its level of support but the potential rival *does not*. The incumbent first chooses how much soft money to spend on the campaign financing: a low quantity (L) or a high quantity (H) *which is observed* by the potential rival. The rival can then decide to run (R) or not to run (N).

If the incumbent chooses a level of campaign financing of L, then given the support base and the reaction of the potential rival, the payoffs are given by the following *payoff matrix* (payoffs represent the expectations from winning, campaigning, etc.):

| | | rival's reaction | |
|---|---|---|---|
| | | **R** | **N** |
| Base of | **B** | **6,4** | **10,0** |
| support | **S** | **4,4** | **6,0** |

The added cost **in payoffs** that an **incumbent** incurs for choosing H instead of L is 2 if he has a broad base of support and 4 if he has a small base of support. A rival that runs against an incumbent with a broad base of support who chose H will obtain a payoff of -10, while a rival that runs against an incumbent with a small base of support who chose H will obtain a payoff of -4. If the rival chooses not to run then he obtains 0 as in the payoff matrix above.

4 (a) Draw the game-tree that represents the extensive form of this game, and identify the proper subgames.

6 (b) Draw the matrix that represents the normal form of the extensive from you did in (a) above. Be clear as to your choice of strategy spaces for each player.

4 (c) If the rival could **commit in advance** to a certain pure strategy that he will follow regardless of the incumbent's choice of financing, **anticipating** that the incumbent will then choose his best response, what would this strategy be? What would be the incumbent's best response to this strategy?

4 (d) Are the pair of strategies you found in (c) above a Bayesian Nash Equilibrium? Explain.

4 (e) Can the pair of strategies you found in (c) above be part of a Perfect Bayesian Equilibrium?

4 (f) Are there other pairs of strategies that can be part of a Perfect Bayesian Equilibrium?

**Question 4: alternative auctions** (24 points)

Two bidders face an auctioneer who plans on selling them an object. Each bidder $i$ has a valuation of either 2, or 4, each of these with equal probability. The valuations of the bidders are independent of each other. Assume that each bidder can **only bid non-negative <u>integer</u> numbers**.

The auctioneer is considering selling the object via a second-price sealed-bid auction: the highest bidder wins, and pays the second highest price. In case of a tie, the first and second highest bids are the same, and the object is given to each bidder with equal probability.

2 (a) Define a pure strategy for player $i$.

4 (b) Show that it is a weakly dominant strategy for each bidder to bid his true valuation.

2 (c) Is it a Bayesian Nash Equilibrium for each bidder to bid his true valuation?

4 (d) Are there other pure strategies that form a Bayesian Nash Equilibria that the bidders would jointly prefer playing?

Now suppose that the auctioneer is considering a first-price sealed-bid auction: the highest bidder wins, and pays his bid. (In case of a tie the good is allocated to each bidder with equal probability.)

4 (e) Is it a Bayesian Nash Equilibrium for each bidder to bid his true valuation?

4 (f) Show that it is a Bayesian Nash Equilibrium for each bidder to bid half of his true valuation.

4 (g) Are there other pure strategies that form a Bayesian Nash Equilibria that the bidders would prefer playing?