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Econ 160: Final Exam

Winter Quarter, 2003

Date: **Monday, 3/17/03** Starting time: **3:30 PM** Ending time: **6:30 PM**

DIRECTIONS (Read Carefully!!):

- This exam includes **4 questions**. Please answer each question in a **separate blue book**. I have tried to order the questions according to their difficulty. It is strongly recommended to read the whole exam before you attempt to solve it.
- Please hand in your answers in a comprehensible format; **illegible answers may lose valuable points**! Also any answer that is not supported by calculations or justifications (when relevant) will receive reduced credit!

GOOD LUCK!!

Question 1: Basic Concepts (10 points)

Consider a 2-player game of *complete information*, in which each player has a weakly dominant strategy denoted by s_i^* . For each of the following statements, provide a proof if it is true or a counter-example if it is not:

4 (a) The strategy pair (s_1^*, s_2^*) is a Nash equilibrium.

6 (b) There cannot be another Nash equilibrium $(s_1^{'}, s_2^{'})$ that gives each player a higher utility than he gets from (s_1^*, s_2^*) .

Question 2: Who Learns Where? (30 points)

Consider the following problem of educational choice. A young adult, player 1, is deciding whether to go to Bear University (B) or to Tree University (T). The difference is that Tree University involves harder work, and imposes more "studying" costs on the player, but at the same time a player who goes to Tree University learns more and is more productive. The costs of learning and the final productivity depends on the type of player 1, who can be either really good (G) or just above average (A). Player 1 knows his type, but all other people in society only know that a proportion p of young adults are G-types. The cost and productivity from each choice is given as follows:

G-type

choice	cost	prod
B	0	4
T	2	12

A-type

choice	cost	prod
B	2	2
T	8	10

Once player 1 finishes school, he is hired by a firm (player 2) who can place player 1 in one of two jobs: low-tech (L) or high-tech (H). The wage for the L job is $w_L = 2$ and for the H job is $w_H = 6$. The payoffs to player 1 are the **wages less the cost of education**.

The firm's profits depend both on the job assignment, and on the type of employee. If the employee is assigned to a H job, the net profits to the firm are equal to **the productivity** of the employed player 1 **less the wage** he is paid. If the assignment is to a L job, the net profits are **half the productivity** of the employed player 1 **less the wage** he is paid.

8 (a) Draw this game in extensive form

8 (b) Assume that $p = \frac{1}{2}$. Represent the matrix form of the Bayesian Game.

4 (c) Find all the pure-strategy Bayesian Nash equilibria.

8 (d) Find all the pure strategy Perfect Bayesian equilibria.

2 (e) What is the brief intuition that explains the comparison between your results in **(d)** and **(c)** above?

Question 3: Reap and Weep (30 points)

A farmer owns some land which he can farm to produce crops. Farming output depends on the talent of the farmer. The farmer knows his talent, but the rest of the world only knows that a farmer's talent is uniformly distributed: $t \in [0, 1]$. The farmers payoff from farming his land is equal to his talent t.

Before setting up his farm, the farmer approaches to the local manufacturing plant and offers to work on the production line. The farmer can ask the plant owner for any wage $w \geq 0$, and the owner can reject (R) or accept (A) the offer. If the owner rejects the offer then the farmer must return home and settle with his farming. If the owner accepts the offer then the farmer's payoff is the wage w, while the owner's payoff is given by the net value $\frac{3}{2}t - w$, and this is common knowledge.

2 (a) Is this a game of complete or incomplete information?

4 (b) Define the set of pure strategies for each player.

10 (c) Find the pure-strategy Bayesian Nash equilibria of this game.

4 (d) Averaging over the type of farmer, what are the possible levels of *social surplus* (sum of expected payoffs of the farmer and the owner in their potential relationship) from the equilibria you derived in (c) above?

10 (e) A local policy-maker who is advocating for the increase of social surplus is proposing to cut water subsidies to the farmers, which would imply that a farmer of type t would get a payoff of $\frac{1}{2}t$ from farming at home. This policy has no effect on the productivity of manufacturing. Using the criterion of social surplus, can you advocate for this policy-maker using equilibrium analysis?

Question 4: Diluted Happiness (30 points)

Consider a relationship between a bartender and a customer. The bartender serves bourbon to the customer, and chooses $x \in [0, 1]$, which is the proportion of bourbon in the drink served, while $1 - x$ is the proportion of water. The cost of supplying such a drink (standard 4 once glass) is $c \cdot x$ where $c > 0$.

The Customer, without knowing x, decides on whether or not to buy the drink at the market price p. If he buys the drink, his payoff is $v \cdot x - p$, and the bartender's payoff is $p - c \cdot x$. Assume that $v > c$, and all payoffs are common knowledge. If the customer does not buy the drink, he gets 0, and the bartender gets $-(c \cdot x)$. Since the customer has some experience, once the drink is bought and he tastes it, he learns the value of x, but this is only after he pays for the drink.

2 (a) Is this a game of perfect or imperfect information?

4 (b) As best as you can, draw the game tree of this game.

5 (c) Find all the Nash equilibria of this game.

4 (d) Now assume that the customer is visiting town for 10 days, and this "bar game" will be played for each of the 10 evenings that the customer is in town. Assume that each player tries to maximize the (non-discounted) sum of his stage payoffs. Find all subgame-perfect equilibria of this game.

10 (e) Now assume that the customer is a local, and the players perceive the game as repeated infinitely many times. Assume that each player tries to maximize the discounted sum of his or her stage payoffs, where discount rate is $\delta \in (0, 1)$. What is the range of prices p (expressed in the parameters of the problem) for which there exists a subgame-perfect equilibrium in which everyday the bartender chooses $x = 1$ and the customer buys at the price p?

5 (f) For which values of δ (expressed in the parameters of the problem) can such a price range that you found in (5) above exist?