Winter 2003 Prof. Steve Tadelis

Econ 160 - Midterm Solutions

Question 1. Basic Concepts (20 points)

Part a

A player's *best response* to a given opponent strategy profile is any of that player's strategies which yield the highest payoff given that opponent strategy profile. Mathematically, s_i is a best response to s_{-i} if

$$U_i(s_i, s_{-i}) \geq U_i(s_i', s_{-i}) \quad \forall s_i' \in S_i$$

Part b

A Nash Equilibrium is a strategy profile where every player is playing a best response to the other players' strategies. Mathematically, s^* is a Nash Equilibrium if

$$U_i(s_i^*, s_{-i}^*) \geq U_i(s_i', s_{-i}^*)$$

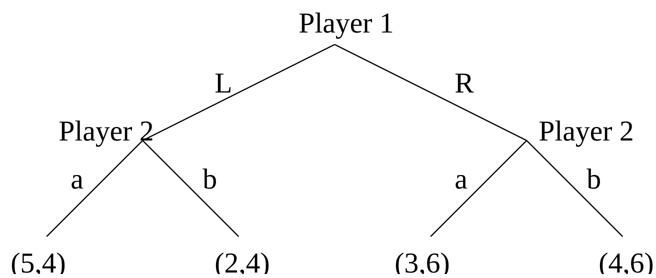
$\forall i$ and $\forall s_i' \in S_i$.

Part c

False. In many games (including games with multiple equilibria, games with mixed-strategy equilibria, and games with no strictly dominated strategies for either player), IESDS will not lead to a pure-strategy Nash equilibrium. One specific example is Matching Pennies - since no strategy for either player is strictly dominated, IESDS does not give any restrictions on what strategies can be played.

Part d

False. *Most* games of perfect information will have a single subgame perfect equilibrium. However, games where two strategies give a player the same payoff could have multiple SPEs. For a nontrivial example, consider the two-player game represented below. In this game, (L,aa), (L,ab), (R,ba), and (R,bb) are all SPE. For an even simpler example, take a one-player game with two moves, L and R, which give the same payoff - either move, or any mix of the two, is an SPE.



Part e

True. This can be proved directly or by contradiction. The proof by contradiction goes as follows:

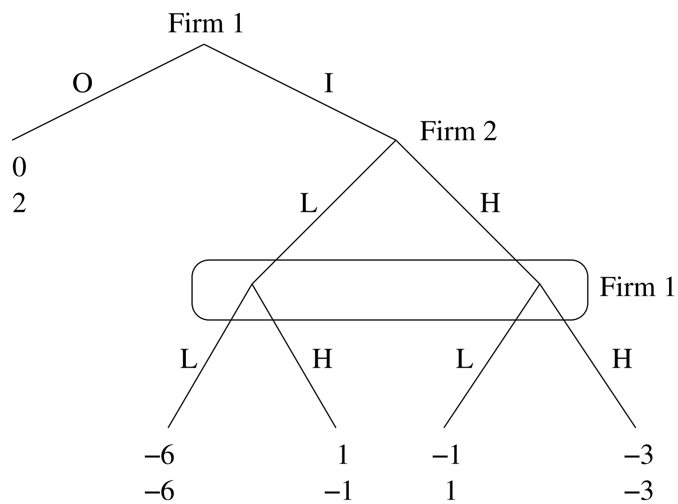
Suppose the statement was false, i.e., there exists a game with a Strict Dominant Strategy Equilibrium that is not a Nash Equilibrium. Let $\tilde{s}$ denote this DSE. Since $\tilde{s}$ is not a Nash Equilibrium, there is some i and some s_i' such that

$$U_i(\tilde{s}_i, \tilde{s}_{-i}) < U_i(s_i', \tilde{s}_{-i})$$

But then $\tilde{s}_i$ is not a Strictly Dominant Strategy, contradicting the assumption that $\tilde{s}_i$ is played as part of a Dominant Strategy Equilibrium.

Question 2. Brand Location (25 points)

Part a



There are two proper subgames: the entire game, and the subgame beginning at firm 2's move following I.

Part b

		Firm 2 L	Firm 2 H
Firm 1	OL	0,2	0,2
	OH	0,2	0,2
	IL	-6,-6	-1,1
	IH	1,-1	-3,-3

Part c

The subgame following I has two pure-strategy equilibria, each of which will lead (by backward induction) to an SPE for the entire game.

First, take the equilibrium where firm 1 plays H and firm 2 plays L. Payoffs are $(1, -1)$. Going back to the top, firm 1 now chooses between O, which gives a payoff of 0, and I, which gives a payoff of 1. The SPE is (IH, L).

Second, take the equilibrium where firm 1 plays L and firm 2 plays H. Payoffs are $(-1, 1)$. This time, firm 1 prefers O, so the SPE is (OL, H).

Part d

The simultaneous subgame has a mixed-strategy Nash equilibrium where both firms play L with probability $\frac{2}{9}$ and H with probability $\frac{7}{9}$. (As always, we find this by assuming firm 1 plays L with probability p and equate firm 2's payoffs to solve for p, then assume firm 2 plays L with probability q and equate firm 1's payoffs to solve for q.) This leads to payoffs of $(-\frac{19}{9}, -\frac{19}{9})$, so firm 1 prefers not to enter. The SPE is then $((O, \frac{2}{9}L + \frac{7}{9}H), \frac{2}{9}L + \frac{7}{9}H)$.

(Mixed strategies are played in the subgame, but the subgame is off the equilibrium path.)

Question 3. Agenda Setting (25 points)

Part a

This is a game of perfect information, as player 2 learns what player 1 has done before he moves. (Put another way, every information set is a singleton.)

Part b

To write down a game in normal form, we must specify *Players*, *Strategies*, and *Payoffs*.

The Players are the Agenda Setter (1) and the Legislator (2).

Player 1's strategies are the possible values between 0 and 5, that is, $S_1 = X = [0,5]$.

Player 2's strategies are the possible plans for what values of x he will 'accept' and for what values of x he will instead choose the status quo. That is, each of player 2's strategies is a map from the interval $[0,5]$ to the set $\{Accept, Reject\}$, so S_2 is the set of functions from $[0,5]$ to this set.

Let $s_2(x)$ denote player 2's choice under strategy s_2 given player 1's move x. That is, $s_2(x) = x$ or 4. Then the payoff functions are:

$$u_1(s_1, s_2) = \begin{cases} 10 - |s_1 - 1| & \text{if } s_2(s_1) = s_1 \\ 7 & \text{otherwise} \end{cases}$$

$$u_2(s_1, s_2) = \begin{cases} 10 - |s_1 - 3| & \text{if } s_2(s_1) = s_1 \\ 9 & \text{otherwise} \end{cases}$$

Part c

From player 2's payoff function, we know that 2 strictly prefers x to 4 when x is (strictly) between 2 and 4, and strictly prefers 4 to x when $x < 2$ or $x > 4$. Given sequential rationality, then, player 2 must play some strategy s_2 such that

$$s_2(s_1) = \begin{cases} s_1 & \text{if } s_1 \in (2,4) \\ 4 & \text{if } s_1 < 2 \text{ or } s_1 > 4 \end{cases}$$

Player 2 can do "anything" given $s_1 = 2$ or $s_1 = 4$. It turns out, however, that a Subgame Perfect Equilibrium only exists if player 2 plays a strategy where $s_2(2) = 2$. (If player 2 chooses the status quo with positive probability given $s_1 = 2$, then player 1's payoff function is discontinuous at 2 and player 1 does not have a best response to s_2.) Therefore, we will consider only strategy profiles where $s_2(2) = 2$.

Knowing that $s_2(2) = 2$ and $s_2(x) = 4 \forall x < 2$, player 1 chooses $s_1 = 2$. Thus, the SPE is $s_1^* = 2$ and

$$s_2^*(s_1) = \begin{cases} s_1 & \text{if } s_1 \in [2,4) \\ 4 & \text{if } s_1 < 2 \text{ or } s_1 > 4 \\ \text{anything} & \text{if } s_1 = 4 \end{cases}$$

It is unique, except that player 2's strategy may vary at the off-equilibrium-path point $s_1 = 4$.

Part d

One Nash equilibrium that is not subgame perfect is $s_1^* = 2.5$ and

$$s_2^*(s_1) = \begin{cases} s_1 & \text{if } s_1 \geq 2.5 \\ 4 & \text{otherwise} \end{cases}$$

It is easy to verify that each player is best-responding to the other's strategy. Similar equilibria can be found by replacing 2.5 by any number s between 2 and 4.

Part e

No, the non-subgame perfect equilibria are not unique. However, *all* Nash equilibria will be of one of the following two types:

Type 1. Equilibria Where $s_2(s_1) = s_1$

These equilibria will all be of the form

$$s_1^* = s, \quad s_2^*(s_1) = \begin{cases} s_1 & \text{if } s_1 = s \\ 4 & \text{if } s_1 < s \\ \text{anything} & \text{if } s_1 > s \end{cases}$$

for some $s \in [2, 4]$.

Type 2. Equilibria Where $s_2(s_1) = 4$

These equilibria will all be of the form

$$s_1^* = s, \quad s_2^*(s_1) = \begin{cases} 4 & \text{if } s_1 = s \\ 4 & \text{if } s_1 < 4 \\ \text{anything} & \text{if } s_1 \geq 4 \end{cases}$$

for some $s \geq 4$ or ≤ 2.

Question 4. Wasteful Shipping Costs? (30 points)

Part a

Each firm acts as a monopolist within its own country, and maximizes profit function

$$\pi_i(q_i^i) = q_i^i(p^i - c_i) = q_i^i(90 - q_i^i - 10) = 80q_i^i - (q_i^i)^2$$

Taking the first-order condition gives

$$\frac{\partial \pi_i}{\partial q_i^i} = 80 - 2q_i^i = 0 \longrightarrow q_i^i = 40$$

so the equilibrium quantities will be

$$q_1^A = q_2^B = 40 \text{ and } q_1^B = q_2^A = 0$$

Part b

Again, we specify *Players*, *Strategies*, and *Payoffs*.

Players are firms 1 and 2.

Strategies are quantity pairs (q_i^A, q_i^B), so $S_1 = S_2 = \mathbf{R}_+^2$.

Payoffs for each firm are profits, which are

$$\pi_i(q_i^i, q_i^j, q_j^i, q_j^j) = q_i^i(90 - q_i^i - q_j^i - 10) + q_i^j(90 - q_i^j - q_j^j - 20)$$

where the 20 in the last term represents production costs and shipping costs.

Part c

Firm i maximizes

$$\pi_i = q_i^i(80 - q_i^i - q_j^i) + q_i^j(70 - q_i^j - q_j^j)$$

Taking first-order conditions given

$$\frac{\partial \pi_i}{\partial q_i^i} = 80 - 2q_i^i - q_j^i = 0, \quad \frac{\partial \pi_i}{\partial q_i^j} = 70 - 2q_i^j - q_j^j = 0$$

and best-response functions

$$q_i^i = \frac{80 - q_j^i}{2}, \quad q_i^j = \frac{70 - q_j^i}{2}$$

We then simultaneously solve

$$q_1^A = \frac{80 - q_2^A}{2}, \quad q_1^B = \frac{70 - q_2^B}{2}, \quad q_2^A = \frac{70 - q_1^A}{2}, \quad q_2^B = \frac{80 - q_1^B}{2}$$

Solving these gives the equilibrium quantities $q_1^A = q_2^B = 30$ and $q_2^A = q_1^B = 20$.

The Nash equilibrium is unique, since best-responses are unique and the best-response functions cross at a single point.

Part d

To do this, we calculate firm 1's profit in the current equilibrium, and in the hypothetical equilibrium which would follow disclosure.

In the current world, the equilibrium quantities found above yield prices of $90 - 30 - 20 = 40$ in both countries, so firm 1's profit is

$$\pi_1 = 30(40 - 10) + 20(40 - 20) = 900 + 400 = 1300$$

(Firm 2's profit is the same, but this is not important.)

Suppose shipping costs jumped to $40q_i^j$. This would give profit functions

$$\pi_i = q_i^i(80 - q_i^i - q_j^i) + q_i^j(40 - q_i^j - q_j^j)$$

and best-responses

$$q_i^i = \frac{80 - q_j^i}{2}, \quad q_i^j = \frac{40 - q_i^j}{2}$$

Equilibrium is then the solution to the simultaneous equations

$$q_1^A = \frac{80 - q_2^A}{2}, \quad q_1^B = \frac{40 - q_2^B}{2}, \quad q_2^A = \frac{40 - q_1^A}{2}, \quad q_2^B = \frac{80 - q_1^B}{2}$$

which turns out to be $q_1^A = q_2^B = 40$ and $q_1^B = q_2^A = 0$. That is, with shipping costs of 40 per unit shipped, it is no longer worth it to export coal, so each firm again acts as a monopolist within its own country. Profits are then

$$\pi_1 = \pi_2 = 40(90 - 40 - 10) = 1600$$

Since firm 1 would get higher profits ($1600 > 1300$) if shipping costs rose to 40, we assume firm 1 would be willing to release the information to the WTO.