

## **A Liquidity-Based Theory of Closed-End Funds**

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### **ABSTRACT**

This paper develops a rational, liquidity-based model of closed-end funds (CEFs) that provides an economic motivation for the existence of this organizational form: They offer a means for investors to buy illiquid securities, without facing the potential costs associated with direct trading and without the externalities imposed by an open-end fund structure. Our theory predicts the patterns observed in CEF initial public offerings (IPOs) and the observed behavior of the CEF discount, which results from a tradeoff between the liquidity benefits of investing in the CEF and the fees charged by the fund's managers. In particular, the model explains why IPOs occur in waves in certain sectors at a time, why funds are issued at a premium to net asset value (NAV), and why they later usually trade at a discount. We also conduct an empirical investigation, which, overall, provides more support for a liquidity-based model than for an alternative sentiment-based explanation.

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# 1 Introduction

A closed-end fund (CEF) is a publicly traded firm that invests in securities. While investors can, in principle, trade either in the CEF's shares or directly in the underlying securities, a CEF rarely trades at a price equal to the value of the securities it holds (its Net Asset Value, or NAV). CEFs usually trade at a discount to NAV, though it is not uncommon for them to trade at a premium. The existence and behavior of this discount, commonly referred to as the "closed-end fund puzzle," poses one of the longest standing anomalies in finance: Why do CEFs generally trade at a discount, and why are investors willing to buy a fund at a premium at its IPO, knowing that it will shortly thereafter fall to a discount?

These considerations have led most authors to conclude that investor irrationality is the only possible explanation. For example, Lee et al. (1991) observe that "it seems necessary to introduce some type of irrational investor to be able to explain why anyone buys the fund shares at the start . . ." Pontiff (1996) concludes that "Pricing theories that are based on fundamentals have had very little, if any, ability to explain discounts," and Chay and Trzcinka (1999) conclude that, "The investor sentiment hypothesis of the formation of closed-end funds appears to be the only plausible explanation for the initial public offering . . ." This leads to a further, even more fundamental question: Do CEFs exist primarily to exploit investor irrationality, or is there another reason for their existence?

In this paper, we provide a simple economic explanation for the existence of CEFs, motivated by the observation that CEFs tend to hold illiquid securities, while their shares are relatively liquid. Investors who trade illiquid assets directly incur potentially large transaction costs. On the other hand, if investors trade the assets indirectly, by buying or selling the relatively liquid shares of an exchange-traded CEF, the underlying illiquid assets do not change hands, and the investors avoid these large illiquidity costs.<sup>1</sup>

In our model, the value of a closed end fund is equal to

$$\text{NAV} + \text{capitalized liquidity benefits} - \text{capitalized manager's fees.}$$

The premium or discount at which a CEF trades therefore emerges naturally from the trade-off between the fund's liquidity benefits and the fees charged by its management, without needing to appeal to investor irrationality. In the absence of fees, the CEF will trade at a premium to NAV. With fees, the CEF may trade at either a discount or a premium, depending on the size of the fees relative to the liquidity benefit. Moreover, the discount will vary

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<sup>1</sup>Unlike other organizational forms, such as open-end funds (OEFs) and exchange-traded funds (ETFs), CEFs are not subject to large-scale creation or redemption of shares, allowing them to better manage their trading in illiquid securities. We discuss these advantages in detail in Section 3.4.

over time with the liquidity difference between the CEF shares and its underlying assets.<sup>2</sup>

Our model not only provides a simple rational explanation for the discount on CEFs, but also makes predictions about their IPO behavior, and sheds light on the behavior of the discount at and after a fund's IPO. In the model, new funds come to market when the (endogenously determined) premium on existing CEFs reaches the level where investors are indifferent between buying a seasoned fund at a premium, or paying a premium for an identical newly IPOed fund; this premium should be high enough to compensate for the underwriters' fees. Thus, IPO investors in our model pay the underwriters' fees not because they are irrational, but because they are interested in the liquidity services provided by a CEF, and these services are currently trading at a high price. The entry of a new CEF effectively decreases demand for the services of other CEFs in that sector, and thus puts downward pressure on the CEF premium, much as the entry of producers into a product market places downward pressure on commodity prices. An equilibrium is characterized by mean-reversion in the premium and, consistent with empirical observation (see Lee et al., 1991; Sharpe and Sosin, 1975), investors buy the fund even though they expect that the premium will subsequently decline. The model also predicts that we should see IPOs occurring in waves in different sectors, since if the liquidity premium in a given sector is high for one fund contemplating coming to market, it will also be high for other funds in the same sector.

A further contribution of this paper is the construction of a comprehensive data set covering US CEFs in existence between 1986 and 2004. This data set allows us to calibrate the model, reexamine previously documented stylized facts using substantially more data, and explore the extent to which the model is qualitatively and quantitatively consistent with reality. It also allows us to contrast our explanation with alternative explanations such as the sentiment model of Lee et al. (1991). On balance, the data do not support the predictions of a sentiment-based model, but do support both the liquidity tradeoffs underlying our model and its predictions for the behavior of CEF discounts. Specifically, we find that (i) the majority of closed-end funds specialize in illiquid securities such as municipal, corporate and international bonds, while CEFs are, themselves, relatively liquid; (ii) consistent with the theory, the CEF premium is negatively related to the manager's fee and the degree of CEF share illiquidity, while it is positively related to the fund's payout and leverage, (iii) both the CEF premium and number of IPOs are related to systematic variables measuring the liquidity benefits provided by the fund; (iv) there is no consistent evidence that investor sentiment measures are positively related to the CEF premium or the number of IPOs; (v)

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<sup>2</sup>Empirically, because at least a portion of liquidity is sector-specific, changes in the discount can be expected to be more correlated between funds of the same type, and less correlated between funds in different sectors (a pattern observed by Lee et al., 1991).

for realistic parameter values, the model is able to match the times-to-discount and average premia observed in the data.

Despite the success of the theory in accounting for most of the key stylized facts, there are some discrepancies. First, our model predicts that the returns on new CEFs will be comparable with those on seasoned CEFs managing similar assets. After controlling for leverage, we find (contrary to prior studies that do not control for leverage) that this is indeed the case for domestic and foreign equity funds. However, stock returns on newly issued muni and taxable fixed income CEFs tend to significantly under-perform those of matched seasoned CEFs. Another, perhaps related, inconsistency is the fact that, while IPOs take place in waves when premia are high, and are somewhat correlated with measures of liquidity benefits (both as predicted by the model), the prevailing premium on seasoned funds during an IPO wave is typically lower than the cost of an IPO.

There are other CEF models in which investors earn a fair rate of return despite the predictable behavior of the discount. The closest is Berk and Stanton (2007), in which the behavior of the discount results from a tradeoff between managerial ability and fees, rather than our tradeoff between liquidity and fees.<sup>3</sup> Unlike ability-based models, our model can explain the patterns observed in CEF IPO behavior, and why discounts on related funds tend to move together. However, the models are complementary. Our explanation does not rule out the existence of managerial ability, and, in principle, we could include both features in a single model. Spiegel (1999) considers a frictionless overlapping generations economy in which agents have finite lives. His economy supports a self-fulfilling beliefs equilibrium in which zero-payoff portfolios sell for nonzero prices, implying that a CEF need not trade at its NAV. He does not, however, explicitly model IPOs or the time-series dynamics of the discount.

The paper is organized as follows: Section 2 gives basic facts about closed-end funds and the behavior of the discount, and motivates the model by discussing the evidence supporting a liquidity rationale for the services provided by CEFs. Section 3 develops a formal model that implements the ideas laid out in prior sections, and calibrates the model to the data. Section 4 conducts a detailed empirical investigation of the model, focusing in particular on tests that can distinguish the liquidity explanation from alternatives such as the sentiment theory of Lee et al. (1991). Section 5 concludes with a summary of our findings.

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<sup>3</sup>A related model is that of Ross (2002b). He also explains the post-IPO discount via a tradeoff between managerial ability and fees. However, unlike Berk and Stanton (2007), IPO investors in his model do not earn the fair rate of return.

## 2 Closed-End Funds and the Discount

This section documents some stylized facts about closed-end funds and the discount, primarily based on the closed-end fund data described in Appendix A. We separate the funds into five classes, based on their prospectus objectives: Municipal bond, taxable fixed income, domestic equity, foreign equity, and other funds. The taxable fixed income category includes funds whose assets mainly include corporate bonds, though some funds also manage government bonds, mortgages and international bonds. Funds classified as ‘Other’ largely manage convertible preferred stocks and other equity-related high income assets.

Table 1 shows CEF IPOs in the years 1986–2004. It can be seen that the new funds invest primarily in illiquid assets such as municipal bonds, corporate bonds, and foreign securities. Though not included in our sample, real estate – another illiquid asset class – also tends to be held by Real Estate Investment Trusts (REITs), which are similar to closed-end funds. The table also reports the total value of assets as of 2005. On a value-weighted basis, well over 50% of the CEFs are bond funds. Table 1 also makes it clear that IPOs occur in waves, a regularity documented in Lee et al. (1991). These waves occur at different times in different sectors. For example, IPOs of foreign equity funds peaked in 1990, a year in which there was only one taxable fixed income IPO (these peaked two years earlier and two years later, in 1988 and 1992). Similarly, there was a wave of muni and taxable fixed income IPOs between 1999 and 2004, yet during this period there were only four foreign equity CEF IPOs, all in 2003 and 2004.

Panel a. of Table 2 documents equally weighted averages of payout ratios, expense ratios, leverage ratios, underwriting costs, and NAV.<sup>4</sup> The payout ratio, while 6.2% on average, varies substantially across fund types, from 1.9% for foreign equity funds up to almost 9% for taxable fixed income funds. The expense ratio is more similar across fund types, though the standard deviations indicate that within types there is substantial variation across funds. It is clear that, across types (with the possible exception of foreign equity funds), CEFs make substantial use of leverage (usually in the form of issued preferred shares), despite the widespread impression that U.S. closed-end funds do not use leverage.<sup>5</sup> CEFs are in general small, averaging just over \$250 million in NAV.

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<sup>4</sup>The leverage ratio is defined as debt divided by total assets; the NAV is defined as shareholders’ assets; and the payout ratio, expense ratio and underwriting costs are defined as the percentage of NAV paid out to shareholders, managers and underwriters (in the event of an IPO), respectively.

<sup>5</sup>For example, Dimson and Minio-Paluello (2002) state that, “Although only a few U.S. closed-end funds take on any leverage, U.K. closed-end funds more frequently make use of leverage through their own capital structures.”

Year	Muni	Taxable FI	Dom. Equity	For. Equity	Other	Total	
1986	1	1	5	3		10	
1987	7	8	5	7		27	
1988	11	25		5	3	44	
1989	15	9	1	5	2	32	
1990	7	3		17	1	28	
1991	30	3		2	1	36	
1992	57	15	3	9	2	86	
1993	63	28	2	6	3	102	
1994	4	8	5	15	1	33	
1995		1		1		2	
1996			2	1		3	
1997	4	1		1		6	
1998	7	8			4	19	
1999	27	1				28	
2000			1			1	
2001	31	3			2	36	
2002	57	5			11	73	
2003	8	18	6	1	16	49	
2004		11	18	3	11	43	
Total	329	148	48	76	57	658	
2005 NAV (\$B)	63	53	33	13	28	190	
FundAge	median	5.6	8.6	10.2	6.9	3.8	6.7
	sd	4.3	8.4	24.0	6.6	5.7	10.2

Table 1: **CEF IPOs.** This table documents the number of CEF IPOs in various sectors from 1986–2004, using data described in Appendix A.

		<b>Muni</b>	<b>Taxable FI</b>	<b>Dom. Equity</b>	<b>For. Equity</b>	<b>Other</b>	<b>All</b>
		N = 332	N = 187	N = 65	N = 81	N = 60	N = 725
<i>a. Balance sheet or income statement attributes</i>							
<b>PayRatio</b>	mean	5.9	8.8	5.1	1.9	7.4	6.2
	sd	1.3	7.2	11.6	10.0	4.7	6.8
<b>ExpRatio</b>	mean	1.1	1.2	1.5	1.8	1.5	1.3
	sd	0.7	0.5	1.5	0.7	0.9	0.8
<b>Leverage</b>	mean	31%	22%	16%	8%	30%	25%
	sd	13%	15%	15%	9%	10%	15%
<b>IPO costs</b>	mean	5.6	5.9	5.9	6.5	4.8	5.7
	sd	1.0	1.1	1.2	0.7	0.9	1.1
<b>NAV</b>	mean	219	282	489	173	324	257
	sd	231	325	597	182	330	314
<i>b. Premium attributes</i>							
<b>Premium</b>	mean	-3.4	-2.6	-6.2	-7.9	-3.6	-4.0
	sd	6.5	9.1	11.9	16.1	8.3	9.9
<b>Tdisc</b>	mean	1.01	1	0.78	1.08	0.74	0.97
	sd	1.22	1.31	1.26	2.15	0.87	1.35
<b>Correlations between fund premium and average premium in sector:</b>							
	Muni	0.558	0.336	0.078	0.160	0.256	
	Taxable FI	0.308	0.509	0.284	0.170	0.522	
	Dom. Equity	0.149	0.287	0.458	0.203	0.374	
	For. Equity	0.185	0.126	0.144	0.490	0.023	
	Other	0.280	0.355	0.223	0.246	0.663	
<b>Excess premium during IPOs</b>							
<b>IPO weighted</b>	mean	1.24	2.78	3.98	7.12	4.25	
	t-stat	1.57	3.98	2.35	4.06	3.09	
<b>#IPOs&gt;4 screen</b>	mean	1.38	3.21	5.22	8.37	5.40	
	t-stat	2.08	2.74	2.51	3.17	2.80	
<i>c. Fund liquidity attributes</i>							
<b>Trading Cost</b>	mean	0.40%	0.54%	0.51%	0.68%	0.52%	0.49%
	sd	0.18%	0.26%	0.30%	0.52%	0.27%	0.29%
<b>Daily Trades</b>	mean	18	63	64	42	76	42
	sd	25	107	61	36	76	70

Table 2: **CEF statistics.** This table reports panel statistics for different sectors of CEFs, using data described in Appendix A.

## 2.1 The CEF Premium/Discount

The CEF premium is defined by

$$\frac{P_t - \text{NAV}_t}{\text{NAV}_t}, \quad (1)$$

where  $P_t$  is the price of one share of the CEF, and  $\text{NAV}_t$  is the net asset value (NAV) per share. Many researchers refer instead to the CEF *discount*, which is the negative of the premium.

The “closed-end fund puzzle” primarily concerns the predictable behavior of the premium over time: Although closed-end funds are issued at a premium commensurate with their underwriting costs, they typically fall to a discount shortly thereafter. Panel b. of Table 2 documents the equally weighted average time-to-discount for each fund type in our sample, calculated according to the procedure described in Appendix A. It is clear from this table that, although rapid, the speed with which CEFs drop from their initial premium to a discount following their IPO is slower than the 120 days noted by Weiss (1989) in her small sample of (mostly equity) CEFs (see also Levis and Thomas, 1995). In our much larger sample, the fall to a discount takes, on average, closer to one year.

Panel b. of Table 2 also shows the average correlation between the premium on funds of each type (rows) and the average sector premia (columns). All correlations are positive, but it can be clearly seen that, for each fund type, the correlation between a fund’s premium and the average premium in its own sector is higher than the correlation between the fund’s premium and the average premium in different sectors.

To get an indication of the relation between IPO waves and the prevailing premium, Panel b. reports, for each sector, the difference between the average premium during IPOs and the unconditional average. The IPO weighted excess premium uses as weights the fraction of IPOs in the six months leading the premium. The “#IPOs > 4 screen” measure is based on the average premium six-months prior to an IPO wave, where an IPO wave is defined as five or more IPOs in a given six-month period.<sup>6</sup> In all cases, the average sector premium during IPOs is well above the unconditional sector premium, and almost always significantly so. Similar evidence was also noted by Lee et al. (1991) and Cherkes (2003a).

According to Panel b., the prevailing premium during IPOs is significantly lower than the cost of issuing an IPO. This could be because the correlation of a fund’s premium with its average sector premium is far from perfect. While some might take this as evidence of overpricing in the CEF industry, this would only be so if investors earned a lower return

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<sup>6</sup>Lagging the premium relative to the IPOs accounts for the fact that IPOs are planned some time before actual inception. More details on how we calculate the excess premium during IPOs are given in Appendix A.

on investing in newly issued IPOs. In addition to more closely examining the systematic determinants of IPOs in Section 4.3, we test for the return underperformance of newly issued IPOs in Section 4.4.

## 2.2 CEFs and liquidity

Panel c. of Table 2 shows that CEFs are a relatively liquid asset class: Across fund types, one-way trading costs average under half a percent, and the number of trades per day is comparable to that of mid and small capitalization stocks on the NYSE (see Chordia et al., 2005). On the other hand, illiquidity costs associated with trading the assets in which CEFs specialize (see Table 1) are particularly severe for the small investors who dominate CEF clientele.<sup>7</sup> For municipal bonds, Green et al. (2007), corroborated by Harris and Piwovar (2006), provide compelling evidence that intermediaries impose a (one-way) mark-up on small trades (those below \$100,000) averaging 2.5%, with mark-ups of 5% not unusual. A small investor with a horizon of one year would thus face annual trading costs averaging 5% higher than those faced by an institutional investor such as a CEF. This should be compared with panel c. of Table 2, which shows that the one-way cost for municipal bond funds averaged 0.4% over the period studied. An individual investor with horizon < 5 years could thus gain substantially by purchasing municipal bonds indirectly, through a CEF. Qualitatively similar results for spreads on corporate bonds are documented by Edwards et al. (2006), who also find that that corporate bonds trade an average of 1.9 times per day, compared with the 67 trades per day for the average taxable fixed income CEF in our sample documented in Panel c. of Table 2.<sup>8</sup>

While we do not have explicit illiquidity measures for assets held by foreign or domestic equity CEFs, Bonser-Neal et al. (1990) document that many countries impose restrictions on trading by foreigners, making direct purchase of foreign equity particularly expensive for U.S. investors. Moreover, an examination of the 24 domestic equity CEFs that IPOed in 2003–2004 reveals that the typical fund provides high income, invests in illiquid securities, and carries significant leverage.<sup>9</sup> Such extensive use of leverage is an additional, indirect liquidity benefit provided by CEFs.<sup>10</sup>

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<sup>7</sup>Empirical evidence indicates that a large proportion of CEF shares are owned by small investors who trade in small lots (see Weiss, 1989; Lee et al., 1991).

<sup>8</sup>In the one-year sample of municipal bond trades studied by Harris and Piwovar (2006), the average municipal bond traded less than once per *week*.

<sup>9</sup>Looking at recent holdings, 16 of the funds hold illiquid securities, eight specialize in tax advantaged assets, and five use derivatives. The average leverage is about 18%.

<sup>10</sup>The Investment Company Act of 1940 allows CEFs to lever up to 100% of shareholders' assets (usually done through the issuance of preferred shares). Because institutions typically enjoy lower financing costs than individuals, it is less costly for a CEF to hold a levered portfolio than it is for a small investor. In

## 3 The Model: CEFs, Liquidity and Equilibrium

### 3.1 Intuition and Assumptions

The stylized facts presented in the preceding section suggest that CEFs can provide small investors with relatively liquid access to what otherwise would be illiquid assets. Investors can choose to buy illiquid assets directly, incurring costs if they unexpectedly have to leave the market and sell their holdings. Alternatively, they may buy indirectly via a closed-end fund. In the latter case, they can always sell their CEF shares to another investor without the underlying assets needing to be sold, thus avoiding the illiquidity costs. In the absence of fees charged by management, CEFs would, as a result, trade at a premium. With fees, CEFs could trade at either a premium or discount, depending on the relative importance of the liquidity benefits versus the fees, and the premium would vary over time if the size of the liquidity benefits did so. Whether this explanation can generate the magnitude of observed discounts and premia, or their dynamics over time in a competitive industry, is a question that can only be answered with a formal equilibrium model, such as that developed here.

The IPO process in equilibrium has important implications for the liquidity premium on the underlying asset. The more investors are willing to pay for the liquidity services of a CEF, the more attractive it is to pool assets under this structure, but since underwriting a CEF is costly, CEFs will not enter until the CEF premium on existing identical CEFs is enough to cover the underwriting costs. At this point, new funds enter the market via an IPO. As long as no new supply of the illiquid asset is introduced, the new CEFs must acquire their assets from the marginal (or price setting) investor in the illiquid security (see Amihud and Mendelson, 1986). A previously infra-marginal investor, who values the illiquid asset more, now becomes the new marginal investor, and the price of the underlying therefore increases (or, alternatively, the liquidity benefit that CEFs provide decreases). The entry of new IPOs thus exerts downward pressure on the liquidity benefit provided by CEFs, which we model via a negative relation between the liquidity premium and the proportion of the illiquid asset held by CEFs. The equilibrium effect of these IPOs by new funds is to impose an upper reflecting boundary on the liquidity premium process (which also determines how often CEFs trade at a large discount, rather than at a large premium).<sup>11</sup>

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addition, the fund's shareholders enjoy the protection of limited liability, a right that does not extend to an individual who attempts to replicate the fund's portfolio.

<sup>11</sup>The economic story described is reminiscent of Dixit (1989) and agrees with the intuition in Gemmill and Thomas (2002), who state (page 2575) that "The lower bound to the discount . . . [a]rises from the relative ease with which new funds can be issued."

**Modeling Assumptions.** We here state the specific assumptions we make in order to implement the model described at an intuitive level above. These assumptions, of course, necessarily sacrifice some realism for the sake of tractability. We discuss this further in Section 3.4.

1. An illiquid asset pays a continuous dividend at rate  $C_t$ , which follows the (risk-adjusted) process:

$$\frac{dC_t}{C_t} = g dt + \sigma_C dZ_t.$$

The parameters  $g$ ,  $\sigma_C$ , and the instantaneous risk-free rate,  $r > g$ , are assumed constant.

2. The asset earns a liquidity premium,  $\rho_t$ , which is uncorrelated with the growth rate of dividends.
3. Unlike the underlying asset, the CEF is perfectly liquid.
4. As long as the fund is in existence, the management receive a fraction  $k > 0$  of the fund's cash flows.
5. Shareholders can force the liquidation of a fund at a cost  $K \times \text{NAV}_t$ , where  $K \geq k$ . Upon liquidation they therefore receive the current value of the assets, net of costs,  $(1 - K) \times \text{NAV}_t$ .
6. New CEFs can enter as infinitesimal units at a cost  $u \times \text{NAV}_t$ , paid to an underwriter ( $u > 0$ ). Other securitization vehicles (e.g., open-end or exchange-traded funds) are not considered.
7. The management, liquidation and underwriting fees are fixed, and are the same across all funds.
8. The observed liquidity premium on the underlying is  $\rho_t = \rho_t^f q(x_t)$ , where  $x_t \in [0, 1]$  is the proportion of the underlying asset currently managed by CEFs, and  $q$  is a monotonically decreasing function with  $q(0) = 1$  and  $q(1) = 0$ . Here,  $\rho_t^f$  is the liquidity premium in the absence of CEFs, which is assumed to evolve as

$$\frac{d\rho_t^f}{\rho_t^f} = \mu dt + \sigma dW_t.$$

## 3.2 Results

**Market Value,  $P_t$ .** Given the assumptions above, because the CEF is perfectly liquid, its market value is the present value of the fund's cash flows discounted by  $r$ , added to the

present value of proceeds from liquidating the fund at some future date,  $\tau$ :

$$\begin{aligned} P_t &= E_t \left[ \int_t^\tau (1-k)C_{t'} e^{-\int_t^{t'} r dt''} dt' \right] + (1-K)E_t[e^{-r\tau} \text{NAV}_\tau] \\ &= C_t(1-k)E_t \left[ \int_t^\tau e^{-(r-g)(t'-t)} dt' \right] + (1-K)E_t[e^{-r\tau} \text{NAV}_\tau]. \end{aligned} \quad (2)$$

In general, the optimal stopping time at which shareholders exercise their option to liquidate the fund,  $\tau$ , maximizes shareholders' cash flows, and is therefore generally stochastic. A valuation of the fund consists of finding the optimal tradeoff between the value of liquidity service provided by the manager, the cost of management, and the option value of terminating the fund. When  $K \geq k$ , the calculation simplifies because it is never optimal to liquidate the fund, and  $P_t$  can be written in closed form:

$$\begin{aligned} P_t &= C_t(1-k)E_t \left[ \int_t^\infty e^{-(r-g)(t'-t)} dt' \right], \\ &= C_t \times \frac{1-k}{r-g}. \end{aligned} \quad (3)$$

**NAV and the Equilibrium Liquidity Premium.** Given the assumptions above, the net asset value of a fund at time  $t$ ,  $\text{NAV}_t$ , is equal to the expected value of all future gross dividends, discounted at the risk-free rate plus the liquidity premium, i.e.,

$$\text{NAV}_t = E_t \left[ \int_t^\infty C_{t'} e^{-\int_t^{t'} (r+\rho_{t''}) dt''} dt' \right], \quad (4)$$

$$= C_t E_t \left[ \int_t^\infty e^{-\int_t^{t'} (r-g+\rho_{t''}) dt''} dt' \right], \quad (5)$$

where the second equality is a result of our assumption that changes in the liquidity premium,  $\rho_t$ , are uncorrelated with shocks to  $C_t$ . The behavior of  $\rho_t$  depends on the equilibrium entry behavior of CEFs. To derive an equilibrium, we first conjecture a process for  $\rho_t$ , then derive the implications for the NAV given this conjecture, and finally verify that parameter choices for the conjectured process exist such that it does indeed constitute an equilibrium.

Specifically, we posit that  $\rho_t$  follows a reflected geometric Brownian motion process between 0 and some upper barrier,  $\bar{\rho} > 0$ . Given this conjecture, Theorem 1 (below) derives the solution to Eq. (5) in closed form. This is then followed by Theorem 2, which shows that equilibrium entry can be rationalized by the conjectured process, and that there is a unique value  $\bar{\rho}$  at which the CEF value equals the NAV plus the underwriting cost, and at which CEFs enter competitively.

**Theorem 1.** *Along with Assumptions 1–8, assume that  $\rho_t$  follows the reflected Brownian*

motion process

$$\frac{d\rho_t}{\rho_t} = \mu dt + \sigma dW_t, \quad \rho_t \in [0, \bar{\rho}].$$

Then the value for the NAV is given by

$$\begin{aligned} \text{NAV}(\rho_t) &= C_t \times \hat{V}(\rho_t), \quad \text{where} \\ \hat{V}(\rho_t) &= \frac{4}{\sigma^2} U_+(\rho_t) \left( \int_{\rho_t}^{\bar{\rho}} \rho'^{\frac{2\mu}{\sigma^2}-2} U_-(\rho') d\rho' - \frac{U'_-(\bar{\rho})}{U'_+(\bar{\rho})} \int_0^{\bar{\rho}} \rho'^{\frac{2\mu}{\sigma^2}-2} U_+(\rho') d\rho' \right) + \\ &\quad \frac{4}{\sigma^2} U_-(\rho_t) \int_0^{\rho_t} \rho'^{\frac{2\mu}{\sigma^2}-2} U_+(\rho') d\rho', \end{aligned} \quad (6)$$

where

$$\begin{aligned} U_+(\rho_t) &= \rho_t^{\frac{1}{2}-\frac{\mu}{\sigma^2}} I \left( \sqrt{\left(1 - \frac{2\mu}{\sigma^2}\right)^2 + 8\frac{r-g}{\sigma^2}}, \sqrt{\frac{8\rho_t}{\sigma^2}} \right), \\ U_-(\rho_t) &= \rho_t^{\frac{1}{2}-\frac{\mu}{\sigma^2}} K \left( \sqrt{\left(1 - \frac{2\mu}{\sigma^2}\right)^2 + 8\frac{r-g}{\sigma^2}}, \sqrt{\frac{8\rho_t}{\sigma^2}} \right), \end{aligned}$$

and where  $I(\nu, y)$  and  $K(\nu, y)$  are the modified Bessel functions of the first and second kind, respectively.

**Proof:** See Appendix B.

**Theorem 2.** Under Assumptions 1–8, there exists a unique threshold  $\bar{\rho} > 0$  determined as the solution to the equation

$$\frac{1-k}{r-g} = \hat{V}(\bar{\rho})(1+u), \quad (7)$$

and characterizing an equilibrium in which

$$\rho_t = \bar{\rho} \frac{\rho_t^f}{s_t^f}, \quad s_t^f \equiv \max\{\bar{\rho}, \sup_{\tau \leq t} \rho_\tau^f\}. \quad (8)$$

CEFs only enter when  $\rho_t = \bar{\rho}$ , the dynamics of  $x_t$  are given by

$$q(x_t) = \frac{\bar{\rho}}{s_t^f},$$

and the probability of becoming managed by CEFs for an infinitesimal fraction of the  $1 - x_t$  supply of illiquid asset not yet under CEF management is

$$\bar{\rho} \frac{1}{q'(x_t)} d\frac{1}{s_t^f}.$$

**Proof:** See Appendix C.

We note that Eq. (8) specifies reflected geometric Brownian motion with the same drift and volatility parameters as  $\rho_t^f$ . Thus Theorem 2 verifies the conjecture used in Theorem 1, showing that the liquidity premium exhibits endogenous mean-reversion, despite the model's fundamentals not being mean-reverting.

**The CEF Premium and Distribution of the Liquidity Premium.** Substituting into Equation (1), the CEF premium can be written as

$$\frac{1 - k}{(r - g)\hat{V}(\rho_t)} - 1. \quad (9)$$

From the integral formula for the NAV in Eq. (4) and the fact that  $\rho_t$  is reflected Brownian motion, one can immediately infer that  $\hat{V}$  is decreasing in  $\rho_t$ , and that  $\hat{V}(0) = \frac{1}{r-g}$ . This can be used to demonstrate that  $\bar{\rho}$  increases with  $k$  and  $u$ . When  $\sigma = \mu = 0$ , so that  $\rho_t = \rho$ , a constant, Eq. (4) can also be used to show that  $\hat{V} = \frac{1}{r+\rho-g}$ , so the CEF premium is given by  $\frac{\rho(1-k)}{r-g} - k$ .<sup>12</sup> When  $\rho = 0$ , this is equivalent to results obtained by Ingersoll (1976), Gemmill and Thomas (2002) and Ross (2002a). If  $\rho$  is non-zero, the fund trades at a (constant) discount or premium, depending on whether  $\frac{\rho}{r-g}$ , the capitalized liquidity savings, is smaller or larger than  $\frac{k}{1-k}$ , the relative ownership of the manager in the fund's assets.<sup>13</sup> Thus, as noted earlier, the CEF premium reflects a tradeoff between the liquidity benefits of organizing the fund versus the loss of ownership in the underlying asset.

If  $\sigma \neq 0$ , the CEF premium can vary in  $[-k, u]$ , and it is equal to  $u$  at the time of IPO. Subsequent to the IPO, the CEF premium decreases. The rate at which it falls to a discount, and its long-run behavior, crucially depend on the value of  $\bar{\rho}$  and on the stochastic process for  $\rho_t^f$  (i.e.,  $\sigma$  and  $\mu$ ). The following result helps in calculating some of these properties.

**Theorem 3.** *The process  $\rho_t$  is stationary if and only if  $\gamma \equiv \frac{2\mu}{\sigma^2} - 1 > 0$ . Moreover, if it is stationary, then the unconditional cumulative distribution function is given by,*

$$F(\rho) = \left(\frac{\rho}{\bar{\rho}}\right)^\gamma.$$

<sup>12</sup>It is possible, though tedious, to obtain the same results directly from Eq. (6). For instance, in Appendix B we illustrate how, setting  $\mu = 0$ ,  $\hat{V}(\rho_t) \rightarrow \frac{1}{r+\rho-g}$  as  $\sigma \rightarrow 0$ .

<sup>13</sup>The result for constant  $\rho_t \neq 0$  was first obtained by Cherkes (2003b).

For all  $\gamma$ , the expected time,  $T$ , it takes for the liquidity premium to reach a level,  $\rho_0 < \bar{\rho}$ , after an IPO is

$$T(\rho_0) = \frac{2}{\gamma\sigma^2} \left( \ln \left( \frac{\rho_0}{\bar{\rho}} \right) - \frac{1 - \left( \frac{\rho_0}{\bar{\rho}} \right)^{-\gamma}}{\gamma} \right). \quad (10)$$

The expected value of any function of  $\rho_t$ , say  $\psi(\rho_t)$ , calculated at  $t = 0$  when the IPO is assumed to take place, is

$$E[\psi(\rho_t) \mid \rho_{t=0} = \bar{\rho}] = \int_0^\infty \left( 2\sqrt{2}\theta N\left(\theta\sqrt{t} - v\sqrt{\frac{2}{t}}\right) e^{-2\sqrt{2}\theta v} + \frac{2}{\sqrt{\pi t}} e^{-\left(\theta\sqrt{t} - v\sqrt{\frac{2}{t}}\right)^2/2} \right) \psi(\bar{\rho}e^{-\sigma\sqrt{2}v}) dv, \quad (11)$$

where  $\theta \equiv \frac{\gamma\sigma}{2}$  and  $N(\cdot)$  is the standard normal cumulative distribution function.

**Proof:** See Appendix D.

By letting  $\rho_0$  correspond to the level of the liquidity premium at which the NAV equals the CEF value, Equation (10) can be used to calculate the average time it takes a CEF to revert to a discount from the IPO. The expected CEF premium  $t$  years after the IPO can be calculated using Equation (11) as  $E[(P(\rho_t) - \text{NAV}(\rho_t)) / \text{NAV}(\rho_t) \mid \rho_{t=0} = \bar{\rho}]$ .

If  $\rho_t$  is not stationary (i.e.,  $\gamma = 2\mu/\sigma^2 - 1 < 0$ ) then, as  $t \rightarrow \infty$ , the liquidity premium almost surely tends to zero, while the CEF premium tends to  $-k$ . Depending on the magnitude of  $\mu$  and  $\sigma$  this might take a considerable length of time, and  $\rho_t$  might find itself again at  $\bar{\rho}$  even long after the first IPO. Intuitively, this represents a situation in which the illiquidity of an asset is a ‘temporary’ phenomenon, and can be rationalized by the expectation that technological innovation will, in the long-run, reduce clearing costs.

From Equation (10), the half life of  $\rho_t$  from its value at IPO is given by  $T_{\frac{1}{2}} \equiv \frac{1}{\sigma^2\gamma}((2^\gamma - 1)/\gamma - \ln 2)$ . This quantity is increasing in  $\gamma$  and decreasing in  $\sigma$ . To gain some perspective,  $T_{\frac{1}{2}} \approx \frac{0.24}{\sigma^2}$  when  $\gamma = 0$ , so a volatility of 50% or more is required to obtain a half-life shorter than a year. While this may seem high, it is empirically plausible. For instance, such ‘high’ volatility would allow for the liquidity premium to change from 4% to 2% in one year. In other words, the volatility of the liquidity premium might be high in relative terms while the overall *level* of the premium is always below a few percentage points (e.g.,  $\bar{\rho} = 6\%$ ).

The possibility that a CEF might fall from a premium to a discount in a period shorter than a year is often cited as a major challenge to any ‘rational’ economic theory. Equation (10) indicates that, by itself, this is *not* inconsistent with our tradeoff model. Short

reversions and a negative average premium several years after IPO are possible if  $\sigma$  is high and  $\gamma$  is small.

### 3.3 Benchmark Calibration

Theorems 1 and 2 give NAV and CEF values as functions of the underlying parameters. In this section, we calibrate the model, and show that, for each CEF sector, it can qualitatively generate a pattern of CEF behavior resembling what is observed in practice. To parameterize the model, one must specify  $k, u, r - g, \gamma$  and  $\sigma$ .<sup>14</sup> Our approach is to select benchmark parameters and then study the comparative statics. In order to make comparison with often quoted CEF statistics, we map our model parameters to the CEF's expense ratio, payout ratio, and a measure of the expected duration of the premium from the inception of an IPO. Specifically, we make the identifications in Table 3.

Variable	Symbol	Observed Statistic	Benchmark Value
Manager's share of dividends	$k$	$\frac{\text{expense ratio}}{(\text{expense ratio} + \text{payout ratio})}$	0.173
Risk neutral interest rate less growth rate	$r - g = \frac{(1-k)C_t}{P_t}$	payout ratio	0.062
Underwriter's fee	$u$	underwriting costs	0.057
Reversion time to a discount	$T_{disc}$ from (10)	reversion time to discount	0.97 yrs
Average premium	Calculated from (11)	average time-series of premia since fund inception	-0.04
Age of the fund		length of time-series	7

Table 3: **Parameter Identification**

<sup>14</sup>Given our assumption of zero correlation between  $C_t$  and  $\rho_t^f$ , the volatility of the payout process,  $C_t$ , is not relevant, and the drift only appears together with  $r$  in the form  $r - g$ .

The benchmark values correspond to the overall estimates from Table 2. The values specified in the table completely and uniquely pin down all five model parameters. The median fund age in our data is seven years (measured from the IPO year – see Table 1). In calculating an average premium for our data, we therefore calculate

$$\text{average premium} = \frac{1}{7} \sum_{t=1}^7 E\left[\frac{P(\rho_t) - \text{NAV}(\rho_t)}{\text{NAV}(\rho_t)} \mid \rho_{t=0} = \bar{\rho}\right]. \quad (12)$$

The implied values,  $\gamma = -0.28$  and  $\sigma = 1.24$ , must be solved for numerically and simultaneously with  $\bar{\rho} = 0.11$  (from Eq. (7)). The value of  $\rho_t$  for which the premium is zero is calculated to be  $\rho_0 = 0.03$ . The volatility is high, and corresponds to a half-life of approximately two months from IPO. Figure 1 plots  $P(\rho_t)/C_t$  and  $\hat{V}(\rho_t) = \text{NAV}(\rho_t)/C_t$  against

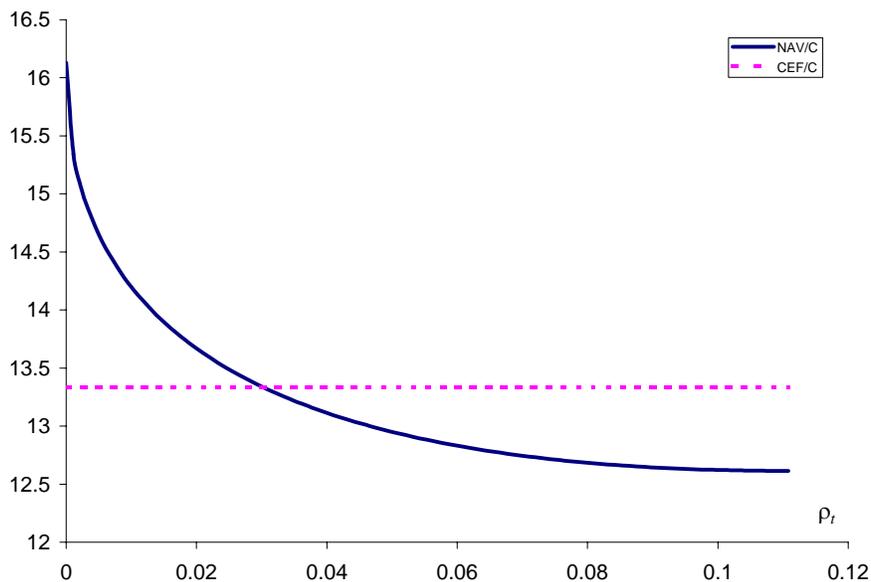


Figure 1: **NAV vs. CEF value.** The solid line shows the NAV (as a multiple of the current cash flow,  $C_t$ ) for different values of the liquidity premium,  $\rho_t$ . The dashed line shows the corresponding CEF value. All parameter values are equal to those given in Table 3.

$\rho_t$ . Figure 2 plots the premium, along with the manager’s fee, as a fraction of the NAV. As expected, for low values of  $\rho_t$ , the CEF trades at a discount, but the discount disappears when  $\rho_t$  reaches 3%, and the fund trades at a premium for higher values of  $\rho_t$ . Figure 2 shows that our assumption of constant  $k$  does not seriously contradict the fact that management fees are usually set to be a fixed proportion of the NAV. Note that the calculated value of  $\gamma$  is negative, meaning that the liquidity premium, for the assets managed by a typical fund,

is not stationary, and is expected to disappear almost surely over time.<sup>15</sup>

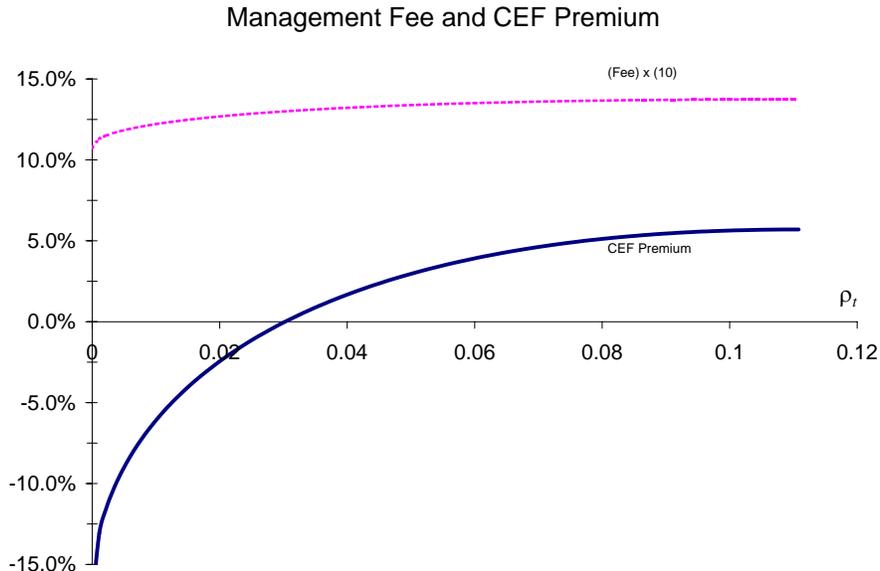


Figure 2: **CEF Premium/Discount vs. Liquidity Premium.** The graph shows the closed-end fund premium and the manager’s fee (as a fraction of NAV) as a function of the liquidity premium,  $\rho_t$ . All parameter values are equal to those given in Table 3.

Figure 3 calculates the expected premium  $t$  years subsequent to an IPO for a fund with the benchmark parameters. The figure also shows the 5% and 95% confidence intervals, indicating that even 15 years out there can be substantial variation in the premium. For comparison, we also plot the data average of the premium using all funds (e.g., for the year 2 point, we tabulate the second year premium for all funds that IPOed during our observation period, and average across the resulting series.) We emphasize that the model graph is fixed by the benchmark parameters in Table 3, and thus we have no extra degrees of freedom to force it to fit the data.

Table 4 shows the calibrated parameter values when the model is fit to the sector data in Table 2. A consistent message from the calibration exercise is that the required liquidity premium volatility must be high for the model to fit the stylized facts, and the underlying process for the liquidity premium is non-stationary in the sense that, eventually, the premium

<sup>15</sup>The lack of ergodicity here, and more generally when  $\gamma \leq 0$ , is a result of our assumed process for  $\rho_t^f$ . It would be possible to specify an alternative model for  $\rho_t^f$  that would allow for ergodicity, yet still generate similar behavior for the discount. For example, letting  $d\rho_t^f = (\rho_t^f \mu + \epsilon)dt + \rho_t^f \sigma dW_t$ , with  $\epsilon \ll \bar{\rho}$ , will lead to an ergodic process for  $\rho_t$  that closely approximates the assumed reflected geometric Brownian motion when  $\rho_t/\bar{\rho}$  is of order 1, but we would lose the tractability of our specification.

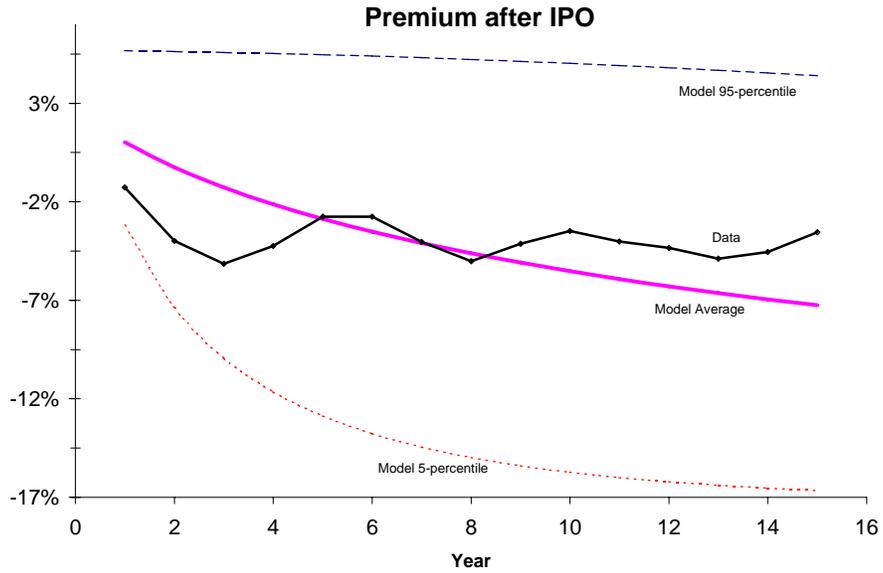


Figure 3: **Distribution of CEF premium after the IPO.** The graph shows the expected premium  $t$ -years subsequent to an IPO for a fund with the benchmark parameters. It also shows the 5% and 95% confidence intervals, and the average premium from the data.

Parameter	Muni	Taxable FI	Dom. Equity	For. Equity	Other	Overall
<i>a. Matched to data</i>						
<b>Payout ratio</b>	0.059	0.088	0.051	0.019	0.074	0.062
<b>Expense ratio</b>	0.011	0.012	0.015	0.018	0.015	0.013
<b>Underwriting costs</b>	0.056	0.059	0.059	0.065	0.048	0.057
<b>Average premium</b>	-0.034	-0.026	-0.062	-0.079	-0.037	-0.041
<b>Time to discount</b>	1.014	1.009	0.792	1.079	0.742	0.975
<b>Fund age</b>	6	9	10	7	4	7
<i>b. Calculated</i>						
$\sigma$	1.160	2.880	1.740	0.360	1.000	1.240
$\mu$	0.426	3.937	1.372	-0.004	0.227	0.556
$\gamma$	-0.367	-0.051	-0.094	-1.063	-0.547	-0.277
$\bar{\rho}$	0.102	0.165	0.147	0.091	0.111	0.111
$\rho_0$	0.029	0.008	0.030	0.061	0.044	0.030

Table 4: **Calibration of the model to the data**

disappears with probability one. Section 4.4 raises the possibility that some of these results might, instead, arise from the overpricing of CEF IPOs.

### 3.4 Discussion of model assumptions

**Assumptions 1–4.** The assumption that the CEF is perfectly liquid is made for simplicity, but does not really affect our results (we could equally well view  $\rho_t$  as the liquidity *difference* between the fund and its underlying assets). The assumption that movements in the liquidity premium are uncorrelated with movements in the growth rate of dividends is made for tractability, as is our assumption that the manager is paid a fixed fraction of the fund’s cash flows. In reality, the manager’s fee is usually set to be a fixed fraction of NAV. As Figure 2 shows, the resulting variation in the manager’s pay as a fraction of NAV is very small.

**Assumption 5.** The liquidation cost,  $K$ , in Assumption 5 reflects both physical expenses as well as less tangible agency costs (e.g., the cost of overcoming a free-rider problem if the fund shares are dispersed). Bradley et al. (2005) discuss the sources of these costs in more detail. The assumption that  $K \geq k$  deters investors from forcing liquidation in equilibrium, and is made for analytic convenience because it allows the model to depend on only a single state variable. If, instead, investors might find it optimal to exercise their option to liquidate a fund, then  $x_t$  (or the supply of illiquid asset) will enter as a second state variable into the dynamics of  $\rho_t$ , and the IPO threshold will vary with  $x_t$ . In the calibration,  $K > k$  implies a liquidation cost of more than 17% of NAV, which is probably on the high side. Allowing for  $K < k$ , and hence endogenous exit, would increase the average CEF premium several years after inception, leading to a better fit between the model and data in Figure 3.

**Assumption 6: CEFs vs. OEFs and ETFs.** There are several reasons why OEFs are less suitable than CEFs for investing in highly illiquid securities: 1. OEF investors face flow-induced trading costs not borne by CEF investors;<sup>16</sup> 2. They also face externalities not shared by CEF investors;<sup>17</sup> 3. OEFs face both legal and self-imposed restriction, not faced by CEFs, on borrowing, trading on margin, short selling, derivative trading, and trade in certain illiquid securities such as privately placed issues (see Almazan et al., 2004). This

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<sup>16</sup>Edelen (1999), in a random sample of equity OEFs, estimates the direct liquidity costs from forced transactions to average 1.5–2% annually.

<sup>17</sup>Among these, Chordia (1996) lists adverse selection costs of trading, brokerage and operating expenses, and unexpected capital gains or losses. He also notes that OEF managers may need to maintain a cash position larger than they would otherwise desire in order to mitigate the impact of redemptions. These externalities create the potential for a fund-run, analogous to a bank-run (see Diamond and Dybvig, 1983; Chen et al., 2007). For example, more than \$32 billion of assets managed by Putnam were redeemed in a single month (see WSJ, 12/8/03). A CEF cannot experience such a fund-run.

is consistent with the empirical results of Deli and Varma (2002), who find that, for both equity and bond funds, the more illiquid a fund’s investments, the more likely is the fund to be a CEF rather than an OEF. It is also consistent with Cherkes (2003b), who finds a smaller turnover ratio and longer duration assets for CEFs vs. OEFs.

Although we do not observe them doing so, ETFs might seem to serve the liquidity needs of small investors without the possibility of trading at a discount to NAV.<sup>18</sup> One would expect investors to be indifferent between ETFs and CEFs, as long as they earn a fair rate of return (as they do in our model). However, there are several possible reasons why CEFs might be preferred in practice: 1. To facilitate redemption and creation of ETF shares, ETFs are based on a prespecified index, limiting their ability to select individual securities to maximize liquidity benefits. 2. Keeping the ETF trading at NAV requires arbitrageurs to trade large amounts of the underlying (illiquid) securities. 3. Underwriters prefer CEFs because they do not receive any compensation for ETF shares created by arbitrageurs. 4. Variation in the size of the fund over time means increased cash flow uncertainty for an ETF manager relative to a CEF manager.<sup>19</sup>

**Assumption 7.** Here we are assuming that anyone who can manage or underwrite a CEF will exhibit the same reservation wage or outside opportunities, and that, subject to this wage, the labor market is perfectly competitive. We note that this assumption is consistent with the absence of heterogeneous skill in the managerial labor market, and that relaxing this assumption entails consideration of a model such as the one explored in Berk and Stanton (2007).<sup>20</sup>

**Assumption 8.** The relation  $\rho_t = \rho_t^f q(x_t)$  is reminiscent of the price-demand relation often postulated in equilibrium models with production (see, for example, Grenadier, 2002).<sup>21</sup>

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<sup>18</sup>An ETF is functionally similar to a CEF, with essentially two major differences: Investors can redeem their shares for the underlying portfolio of assets at any time, and investors also have the right to purchase (directly from the fund company) large blocks of newly issued ETF shares with a basket of securities that mirrors that ETF’s portfolio. This forces an ETF to trade at or near its NAV.

<sup>19</sup>Because CEFs start trading at a premium, ETFs seeking to mirror CEFs would probably start out relatively larger (arbitrageurs would increase the size of the fund until it traded at NAV by creating new ETF shares). Similarly, because CEFs tend to a discount, the ETF would eventually end up relatively smaller (because of the redemption option). The ETF manager’s pay could, on average, be either higher or lower than the CEF manager’s, depending on which of these two effects dominates.

<sup>20</sup>In practice, managing a portfolio of illiquid assets entails skill, albeit not necessarily ‘stock-picking’ or ‘market-timing’ skill. For instance, the manager will have to possess detailed institutional knowledge and/or industry relationships in order to minimize transaction costs when trading in the underlying. Moreover, trading in the underlying is often unavoidable (e.g., a bond fund might replace maturing securities), and their tax treatment is often complicated.

<sup>21</sup>As is customary with equilibrium models positing price-demand relations, Assumption 8 dispenses with any dependence on forward looking variables. The multiplicative form is chosen for tractability.

Some readers may be concerned that the proportion of illiquid assets, such as municipal bonds, held by CEFs is too small for CEF entry to affect the liquidity premium in that sector. To address this, one could modify Assumptions 3 and 8 to assert that the liquidity premium  $\rho_t$  is constant (or exogenously mean-reverting), while the *relative* liquidity benefit provided by CEFs varies with time, and decreases with the entry of new CEFs. The equilibrium premium in such a model would exhibit behavior similar to ours, but the equilibrium analysis would be more involved, and analytically intractable.

Assumptions 8 & 5 and Theorem 2 imply that the proportion of illiquid asset under CEF management can never decrease with time. However, the model can be readily re-interpreted so as to do away with this feature. We provide details in Appendix C, following the proof of Theorem 2.

## 4 Empirical Analysis

In this section, we perform an in-depth empirical analysis, focusing in particular on differentiating between our liquidity explanation for the premium and the most popular alternative, the investor sentiment theory. This was first suggested by Zweig (1973), and modeled by DeLong et al. (1990) and Lee et al. (1991). One fundamental difference between the two explanations is that our liquidity-based theory provides a clear economic rationale for CEFs’ existence—CEFs provide small investors with access to illiquid securities that would otherwise be prohibitively expensive—whereas, in the sentiment models, the existence of CEFs makes investors *worse off*.<sup>22</sup> Consistent with the literature, we take the sentiment hypothesis to mean that the demand of small investors (i.e., the CEF clientele) either depends on factors orthogonal to economic fundamentals, or depends on economic fundamentals in a manner hard to reconcile with “optimizing” behavior.<sup>23</sup>

In Section 3, we interpret  $\rho_t$  as the time-varying liquidity premium on the underlying securities, but we could alternatively interpret  $\rho_t$  as a measure of investor sentiment driving the degree of over- or underpricing of CEFs.<sup>24</sup> While both liquidity and sentiment can give

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<sup>22</sup>As Lee et al. (1991) state (p. 84), “In this theory, then, there is no ‘efficiency’ reason for the existence of closed-end funds. Like casinos and snake oil, closed-end funds are a device by which smart entrepreneurs take advantage of a less sophisticated public.” Similarly, Weiss Hanley et al. (1996) (p. 130) conclude that “. . . the \$1.3B in underwriting fees were an expensive tribute to the informational disadvantage (or irrationality) of small investors.”

<sup>23</sup>We intentionally abstain from a more precise model, such as that offered by Lee et al. (1991), in order to compare our model’s predictions with sentiment models in general, rather than any one specific implementation.

<sup>24</sup>As observed by the referee, we could also postulate a hybrid model, in which liquidity explains the existence of CEFs, but the time series dynamics of the premium/discount are primarily driven by investor sentiment. In this case,  $\rho_t$  would measure the difference between the liquidity and sentiment effects.

Paper	Sectors	Cross- Section	Time- Series	Supports	
				Sentiment?	Liquidity?
Brown and Cliff (2004)	DE		X	No	
Qiu and Welch (2006)	Index: FE+DE		X	No	
Lemmon and Portniaguina (2006)	DE		X	No	
Chan et al. (2005)	FE	X	X		Yes
Bonser-Neal et al. (1990)	FE	X	X		Yes
Amihud et al. (2000)	DE	X			Yes
Datar (2001)	DE, Bonds	X	X		Yes

Table 5: **Existing Research on Determinants of the CEF premium.** In the sector column, Domestic Equity is abbreviated to DE, and Foreign Equity is abbreviated to FE.

rise to similar looking CEF premium behavior, they differ in the underlying factors that drive the premium. One way to distinguish between the two models is therefore to test how much the premium is related to explicit measures of liquidity and sentiment.

Several recent papers test for a relation between the CEF premium and either sentiment measures or liquidity measures. Table 5 summarizes the results of this research, which has found little or no evidence that explicit sentiment measures are related to the premium, but has found some evidence that liquidity-related variables are important. Qiu and Welch (2006) conclude that “In light of our evidence, we believe [the closed-end fund premium] to be inadmissible as a reasonable proxy for investor sentiment.”

While the results of this existing research are consistent, there is nevertheless a need for additional work. First, all of these studies test for either liquidity effects or sentiment effects, but not both. If liquidity and sentiment are correlated with each other, omitting either variable could cause us to find a relation that does not really exist, or to find no relation when one does. Second, most of the studies analyze only a few sectors, and consider either cross-sectional or time-series effects, but not both. Third, the studies do not fully control for individual fund characteristics (e.g., leverage) before aggregating premia across funds. Compared with these prior studies, we analyze many more funds, of more different types, over a much longer time period, looking at both time-series and cross-sectional effects. We also incorporate many more possible explanatory factors into our analysis, including both systematic (market-wide and sector-specific) and fund-specific factors. Finally, unlike prior research, we simultaneously test for the effect of both liquidity and sentiment variables.

## 4.1 Variables

In our empirical analysis, we consider both fund-level and systematic liquidity/sentiment variables that our model (or the sentiment-based alternative) predicts ought to affect the premium.

### 4.1.1 Fund-level variables

**Expense Ratio and Payout Ratio.** Either increasing the payout ratio, *payout*, or lowering the expense ratio, *exprat*, increases the share of the fund’s cash flows that go to the investor rather than the manager. As a result, we expect the premium to be negatively related to the expense ratio, and positively related to the payout ratio in the liquidity model. While the types of sentiment model proposed by DeLong et al. (1990) and Lee et al. (1991) do not make explicit predictions about the relation between fund-specific variables and the fund premium, interpreting  $\rho_t$  in our model as a sentiment variable clearly leads to the same predictions as would be obtained in the liquidity-based interpretation. Data on *payout* are obtained from CRSP, and data on *exprat* are obtained from S&P Capital IQ.

**CEF liquidity.** In our liquidity model, the premium is driven by the relative liquidity of the CEF versus its underlying assets. The higher the CEF’s trading costs, the lower its liquidity advantage relative to the underlying assets. Everything else being equal, we expect the premium to be negatively related to CEF liquidity, for which we use two different measures, the Roll (1984) trading cost measure, *cmdm*, and an estimate of the Pástor and Stambaugh (2003) reversal measure of liquidity, *gamma*. Both measures are obtained from Joel Hasbrouck.

**Fund leverage.** While our model does not incorporate leverage effects, Table 2 indicates that CEFs make intensive use of leverage. It is therefore important in the empirical investigation to account for any possible effects leverage might have on the premium.<sup>25</sup> Data on each fund’s leverage, *lev*, are obtained by interpolating quarterly data from S&P Capital IQ.

### 4.1.2 Systematic variables

**Liquidity measures.** Both the Pástor and Stambaugh (2003) measure of aggregate liquidity, *liq\_level2*, and the Sadka (2006) measure of aggregate illiquidity, *variable\_component*, are priced liquidity factors. Consequently, one might expect the systematic liquidity of domestic

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<sup>25</sup> To get a sense for how leverage might affect the premium, consider an unlevered (all-equity) CEF that provides a premium of  $p_U$  on a unit of assets. An all-equity fund managing  $1+v$  assets is worth  $(1+v)(1+p_U)$  to its shareholders. If the fund subsequently borrows  $v$  (risk-free and liquid) and distributes the proceeds to its shareholders, then the equity (NAV) of the fund falls to 1, while the value of the fund to the shareholders falls to  $(1+v)(1+p_U) - v$ . The levered premium is therefore

$$p_L = ((1+v)(1+p_U) - v) - 1 = p_U(1+v),$$

or, setting  $L \equiv v/(1+v)$  to be the leverage ratio,  $p_U = (1-L)p_L$ . Because  $p_U$  does not depend on leverage, this means that we can control for the effect of leverage by placing  $(1-L)p_L$  on the left side of our regressions.

equity, foreign equity, and the types of assets managed by ‘Other’ CEFs, to be correlated with these variables. As long as one controls for the trading costs of the CEF shares, the liquidity-based explanation predicts that the premium in the Domestic Equity, Foreign Equity, and Other sectors will decrease with *liq\_level2* and increase with *variable\_component*. Data on *liq\_level2* are obtained from WRDS, and data on *variable\_component* are obtained from Ronnie Sadka.

Consistent with Longstaff et al. (2005), we use the yield spread between AAA corporate bonds and Treasury bonds, *corpspread*, obtained from Global Financial Data, as a measure of systematic liquidity in the Taxable Fixed Income sector. Some funds classified as ‘Other’ manage preferred shares, thus we also include *corpspread* as a systematic liquidity factor when analyzing these funds. Because of the tax status of municipal bonds, constructing a measure analogous to *corpspread* for the Muni sector is more involved. We use the Green (1993) formula and muni/Treasury yields from Bloomberg to calculate the implied tax rate for a seven-year muni strip.<sup>26</sup> The federal marginal tax rate on dividend income, obtained from the NBER, is subtracted from the calculated muni-implied tax rate to obtain *taxDiff*. This variable represents a (negative) tax-adjusted yield spread on municipal bonds. Low realizations of *taxDiff* correspond to relatively high tax-adjusted yields on municipal bonds, implying lower liquidity, and consequently a higher CEF premium. A liquidity-based model therefore predicts a negative relation between *taxDiff* and the premium on Muni funds.<sup>27</sup>

Finally, CEFs provide some liquidity benefits via leverage. This involves borrowing at short-term interest rates by issuing preferred shares, while their assets exhibit longer duration. As a result, the advantage to shareholders ought to increase with the slope of the term-structure,<sup>28</sup> so we also include *term*, the difference between the 20 year and 3 month Treasury rates, obtained from Global Financial Data.

**Sentiment measures.** The widely used University of Michigan household sentiment index, *top\_sent*, obtainable from the University of Michigan at <http://www.sca.isr.umich.edu>, is calculated from a regular survey of a large number of households regarding their financial situation and economic expectations. This measure is closely related to other survey-based measures of investor sentiment (Qiu and Welch, 2006; Fisher and Statman, 2003), and has been shown to be related to investor economic activity (Acemoglu and Scott, 1994; Carroll

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<sup>26</sup>The choice of a seven-year strip corresponds to the average duration of municipal bonds held by CEFs in 2000 (obtained from Morningstar).

<sup>27</sup>It is worth emphasizing that, while both *corpspread* and *taxDiff* will be correlated with systematic liquidity in their respective sectors, neither is a direct measure of liquidity.

<sup>28</sup>For an explanation of how the preferred shares dividend yield is determined, see [http://www.nuveen.com/etf/about/preferred\\_overview.aspx](http://www.nuveen.com/etf/about/preferred_overview.aspx). The tax advantages of muni CEFs are passed along to preferred shareholders.

et al., 1994; Bram and Ludvigson, 1998; Ludvigson, 2004). Most important, Lemmon and Portniaguina (2006) find that this measure predicts the returns on small stocks and stocks with low institutional ownership. This is consistent with Fisher and Statman (2003), who find that consumer confidence does not forecast S&P returns, but can predict returns on Nasdaq and small-cap stocks. Because CEFs are small stocks with low institutional ownership, the evidence supports the use of *top\_sent* as a direct measure of the investor sentiment variable described by DeLong et al. (1990) and Lee et al. (1991). If the sentiment interpretation of the model is appropriate, the more optimistic investors are, the higher the CEF premium ought to be, so we should expect a positive relation between *top\_sent* and the premium.

The S&P 500 volatility index, *vix*, obtainable from WRDS, is perhaps one of the more interesting variables, because the two models we are considering have competing implications for its relation with the premium. *vix* is calculated from market prices of CBOE-traded options on the S&P 500 Index. It is often referred to as the market’s “fear gauge” (see Whaley, 2000), and is widely used as a negative measure of investor sentiment. Under the sentiment model, a higher level of *vix*, meaning lower investor sentiment, should translate into a lower premium. On the other hand, both inventory and information asymmetry models predict a positive relation between spreads and volatility (see, for example, Ho and Stoll, 1983; Admati and Pfleiderer, 1988; Foster and Viswanathan, 1990), and empirical evidence supports this prediction (see Stoll, 2000). Thus, under the liquidity model, higher market-wide volatility (i.e., *vix*) should mean lower liquidity for the underlying, and, if one controls for the liquidity of the CEF share, a *higher* premium on the CEF.<sup>29</sup>

Table 6 summarizes the variables used in our analysis, along with the expected relation between each variable and the CEF premium under the liquidity model versus the sentiment model. Table 7 shows summary statistics and correlations for the systematic variables. Interestingly, despite *vix* and the Michigan index, *top\_sent*, both being commonly used measures of investor sentiment, the correlation between them is small (though positive). There is a sizeable correlation between *vix* and the two bond spread measures, as well as between the two bond spread measures. Both the Pástor and Stambaugh (2003) and Sadka (2006) liquidity measures are negatively correlated with *vix*, the correlation between the two variables being small (though positive).

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<sup>29</sup>We stress, however, that because *vix* does not directly measure market liquidity or investor sentiment, one cannot place as much confidence in this measure as, say, the variable *top\_sent*.

Variable Name	Description	Predicted Coeff.	
		Liquidity	Sentiment
a. Fund-Specific:			
<i>exprat</i>	Expense ratio	–	–
<i>payout</i>	Payout ratio (ordinary cash dividends)	+	+
<i>cmdm</i>	Estimate of Roll (1984) trading cost	–	0
<i>gamma</i>	Estimate of Pástor and Stambaugh (2003) reversal measure of liquidity	–	0
<i>lev</i>	Fund’s leverage, interpolated from quarterly data	+	+
b. Systematic:			
<i>liq_level2</i>	Pástor and Stambaugh (2003) liquidity measure	–	0
<i>variable_component</i>	Variable component of Sadka (2006) illiquidity measure	+	0
<i>term</i>	Term spread (20 year – 3 month rate)	+	0
<i>top_sent</i>	U. Michigan Consumer Sentiment Index, top-third income households	0	+
<i>vix</i>	S&P 100 volatility index	+	–
<i>corpspread</i>	Corporate bond spread (AAA corp. bond yield minus Treasuries)	+	0
<i>taxDiff</i>	Spread between the Green (1993) 7-yr muni implied tax rates and the marginal tax rate on dividends	–	0

Table 6: **Variables.** This table describes the variables used in our empirical analysis in Sections 4.2 and 4.3, and shows the expected relation between each variable and the CEF premium under the liquidity model versus the sentiment model. The variables *vix*, *corpspread*, and *taxDiff* are set apart from the remaining systematic variables to indicate that they are more indirect proxies for sentiment or sector liquidity.

Variable:	vix	top_sent	corpspread	taxDiff	term	variablecomp	liq_level2
<i>a. Summary statistics</i>							
mean	19.52	100.09	0.65	0.59	2.26	0.00019	-0.02422
sd	6.41	10.99	0.41	6.03	1.27	0.00468	0.06824
p50	18.97	100.70	0.54	0.69	2.05	0.00086	-0.01595
max	44.28	126.10	1.79	16.14	4.69	0.01102	0.20185
min	10.63	68.60	-0.13	-20.00	-0.25	-0.02081	-0.46154
N	188	240	240	177	240	240	228
<i>b. Correlations</i>							
vix	1.000						
top_sent	0.155	1.000					
corpspread	0.503	0.143	1.000				
taxDiff	-0.578	-0.055	-0.591	1.000			
term	-0.274	-0.447	-0.267	0.287	1.000		
variablecomp	-0.272	0.093	-0.111	0.316	0.000	1.000	
liq_level2	-0.345	-0.085	-0.079	0.226	-0.007	0.184	1.000

Table 7: Summary statistics and correlation matrix for systematic variables.

## 4.2 Determinants of the CEF premium

The large amount of noise in the NAV data poses a problem when trying to assess the relationship between the systematic variables and the fund premia, while controlling for fund specific effects.<sup>30</sup> We therefore use the following three-stage approach, and, after reporting its results, check for robustness with alternative complementary tests.

1. Because the fund-specific factors may, themselves, be correlated with the systematic factors, the first stage of our analysis is to remove the systematic component of the fund-specific variables,  $F_{it}$ , by regressing them against the systematic variables,  $S_t$ ,

$$F_{it} = \beta_{0,i}^F + \beta_i^F S_t + \epsilon_{it}^F, \quad i = 1, 2, \dots, I,$$

where  $I$  is the number of funds. Define  $F_{it}^*$  to be the sample residuals from this regression, i.e. the fund-specific variables stripped of any component related to the systematic variables:

$$F_{it}^* = F_{it} - \hat{\beta}_{0,i}^F - \hat{\beta}_i^F S_t.$$

2. Let  $P_{it}^L$  be the observed (and potentially levered) fund premium. In the second stage, the unlevered premium for each fund,  $P_{it}^{UL} \equiv P_{it}^L(1 - L_{it})$  (see footnote 25), is regressed against  $F_{it}^*$ , i.e., against the component of the fund-specific variables that is *not* related

<sup>30</sup>Comparing two sources of premium data, we found that they agreed on average, but that the discrepancy exhibited a standard deviation of 3%. This gives a sense of the noise present in individual CEF premium quotes.

to the systematic variables:

$$P_{it}^{UL} = \beta_{0,i}^{UL} + \beta_i^{UL} F_{it}^* + \epsilon_{it}^{UL}. \quad (13)$$

Define  $P_{it}^{UL*}$  to be the sample residuals from this regression,

$$P_{it}^{UL*} = P_{it}^{UL} - \hat{\beta}_{0,i}^{UL} - \hat{\beta}_i^{UL} F_{it}^*.$$

Among other things, this second stage regression controls for illiquidity of the CEF shares. The residual,  $P_{it}^{UL*}$ , is the unlevered premium, stripped of fund-specific influences. The coefficients,  $\beta_i^{UL}$ , should, on average, have the predicted signs documented in Table 6.

3. In the third stage,  $P_{it}^{UL*}$  (i.e., the unlevered premia, stripped of fund-specific effects) are aggregated across sectors (to increase power in the time-series),

$$P_t^s = \frac{\sum_{\text{fund } i \text{ is in Sector } s} P_{it}^{UL*}}{\text{No. funds of type } s}.$$

These aggregated premia are then regressed on the systematic liquidity, sentiment, and macroeconomic variables described in Table 6,

$$P_t^s = \beta_0 + \beta S_t + \epsilon_t. \quad (14)$$

Panel a. of Table 8 summarizes the aggregated  $t$ -statistics from the second-stage regressions, Eq. (13) (i.e., the average  $t$ -statistics times  $\sqrt{N}$ , where  $N$  is the number of funds). Consistent with the model, and with the findings of Gemmill and Thomas (2002) and Ross (2002a), the premium is negatively related to the non-systematic expense ratio, and positively related to the non-systematic payout ratio. The liquidity-based model also predicts a negative relation with non-systematic measures of the CEF share illiquidity (*cmdm* and *gamma*). This is essentially borne out (*gamma* is only marginally significant), and is also consistent with the findings of Datar (2001).

Panel a. of Table 8 also documents that the unlevered premium is cross-sectionally and positively related to the non-systematic leverage of the fund. In other words, it appears that the effect of leverage goes beyond the adjustment made in footnote 25. This is consistent with two possibilities: 1. The sharing of the fund's income between the manager and shareholders improves in favor of the shareholders when the fund is levered; or 2. The market 'prices' the fact that it is less costly for the fund than for its clients (i.e., small investors) to hold a levered portfolio. To see which explanation might better fit the data, we calculate the growth in

gross manager pay, and regress this (with fixed effects) against the fund’s change in leverage. The coefficient on leverage is positive and highly significant, meaning that managers enjoy higher pay after an increase in leverage. The positive and significant impact of leverage on the unlevered premium documented in Table 8 is thus due to the liquidity benefits of leverage (which apparently overwhelm the increase in the share of payouts to the manager). We emphasize once more that the liquidity benefit of leverage is not generally provided by OEFs, whose use of leverage is highly restricted.

Overall, the second stage regression explains 72% of the variation in the unlevered premium. While substantial, much of this is due to the large number of parameters being estimated.<sup>31</sup> Results for the third stage, Eq. (14), are shown in panel b. of Table 8. These are broadly consistent with the predictions of the liquidity model, and inconsistent with those of the sentiment model. In three of the four sectors where *vix* is significant, the regression coefficient has the sign predicted by the liquidity model, the opposite of that predicted by the sentiment model. Further evidence against the sentiment model is provided by the other sentiment measure, *top\_sent*, whose coefficient has the opposite sign from that predicted by the sentiment model in every instance in which it is significantly different from zero.

Looking at the sector-specific variables, the *taxDiff* variable has the sign predicted by the liquidity model and is highly significant, and the equity market liquidity variables have the correct signs when they are significant. However, the corporate bond spread coefficient has the wrong (though insignificant) sign for taxable FI funds, while its coefficient in the ‘Other’ CEF premium regression is both highly significant and contrary to our expectations.

If, as suggested by our analysis of the second-stage regression, fund leverage is a cost saving device for those investors who wish to have a levered portfolio, then one would expect these costs savings to increase when borrowing rates for institutions are relatively low.<sup>32</sup> Thus *term*, which we believe proxies for the liquidity benefit of leverage, ought to be positively related to the systematic and unlevered portion of the premium. This is corroborated in Table 8, which documents a positive and significant coefficient for all but one of the *term* coefficients.

**Robustness check** There are several potential issues with the three-stage regression that we now address. First, ignoring the fact that there are differing numbers of funds per period, the results of the third stage ought to be identical to the results of pooling  $P_{it}^{UL}$  by sector,

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<sup>31</sup>Running regressions using only (fund-specific) constants on the right hand side allows us to explain 49% of the variation in the premium.

<sup>32</sup>The CEF industry literature often touts the advantage of leveraging when the yield curve is steep. The regression coefficients remain highly significant and negative when the three-month Treasury rate is substituted for *term* in our regressions.

Variable	Aggregate t-stat	N
<b>lev</b>	2.56	852
<b>exprat</b>	-4.39	852
<b>payout</b>	3.01	852
<b>cmdm</b>	-3.84	852
<b>gamma</b>	1.88	852
<b>const</b>	-20.45	852

*a. Second stage: Fund-specific variables*

	Muni	Taxable FI	Dom. Equity	For. Equity	Other
<b>term</b>	0.553 (3.24)	0.507 (3.14)	0.16 (0.72)	0.375 (1.99)	0.517 (3.19)
<b>corpspread</b>		-0.534 (-1.3)			-2.61 (-6.69)
<b>vix</b>	-0.04 (-1.44)	0.111 (4.43)	0.103 (2.78)	-0.1 (-3.18)	0.123 (4.42)
<b>top_sent</b>	0.022 (1.07)	-0.046 (-2.26)	-0.062 (-2.2)	0.007 (0.3)	-0.085 (-4.17)
<b>variablecomponent</b>			87.862 (1.41)	110.569 (2.1)	136.283 (3.11)
<b>liq_level2</b>			2.652 (0.93)	1.738 (0.72)	-2.71 (-1.31)
<b>taxDiff</b>	-0.123 (-3.44)				
<b>Cons</b>	-2.531 (-1.11)	1.849 (0.77)	3.949 (1.2)	0.873 (0.31)	6.714 (2.81)
<b>Adj R2</b>	0.109	0.3186	0.1386	0.2201	0.6003

*b. Third stage: Systematic variables*

**Table 8: Regression Results** This table show the results of a three stage regression of CEF premium against fund-specific and systematic explanatory factors. In the first stage (not shown), the systematic component of the fund-specific variables is removed by regressing them against the systematic variables. Panel b. shows the aggregate  $t$ -statistics from the second stage, in which the unlevered premium is regressed against the residuals from the first stage regression. Panel c. shows the results of the third stage, where the residuals from the second stage regressions are aggregated across sectors, and regressed on systematic liquidity-related and sentiment-related variables.

and regressing it on  $S_t$  directly (because  $P_{it}^{UL*}$  is equal to  $P_{it}^{UL}$  less something uncorrelated with  $S_t$ ).<sup>33</sup> Running this alternative pooled regression with only systematic variables results in coefficients whose signs and significances are almost always consistent with the results from the third-stage regression reported in panel b. of Table 8.

Another issue is that, because we are using estimated residuals from the first-stage regression, the coefficients from the second-stage regression suffer from an errors-in-variables problem, and the multivariate nature of the estimation makes the direction of the bias difficult to estimate. The result is that we may fail to completely control for the fund-specific effects in the third-stage of the regression. To address this, for each sector we run a mixed-effects panel regression of the unlevered CEF premium, allowing the coefficients (and intercept) to vary across funds for the fund-specific variables, but not for the systematic variables. Here too, we found no substantial difference in the signs or significance of the systematic variable loadings.

Finally, because of the errors-in-variables problem in the second stage of the three-stage regression, we also run a panel fixed-effects regression of the unlevered premium against the fund-specific variables only. Except for *gamma*, which remained insignificant, the coefficients retain the same sign and significance. This is also the case when we run a mixed-effects regression involving all funds, all fund-specific variables, and all systematic variables.

**Summary** Summarizing these results, it is helpful to refer back to the variables listed in Table 6. Looking first at the fund-specific variables, we see that the CEF premium moves in the direction predicted by both the liquidity and sentiment models in response to changes in the expense ratio and payout ratio. The premium also moves with CEF liquidity, as predicted by the liquidity model, and the data support the idea that CEF leverage provides an additional liquidity benefit. Looking next at the systematic variables, we see that the CEF premium generally moves as predicted by the liquidity model in response to changes in the sector-specific liquidity variables (with the possible exception of the corporate bond spread). On the other hand, in almost every case where there is a significant response to changes in the sentiment variables, the response is in almost always the opposite of that predicted by the sentiment model. Thus, on balance, the results of our analysis provide some support for the liquidity model, and provide evidence against the predictions of the sentiment model of Lee et al. (1991), confirming the negative results of Lemmon and Portniaguina (2006), Qiu and Welch (2006) and Brown and Cliff (2004).

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<sup>33</sup>However, under this alternative scheme, if the number of funds is correlated with the systematic variables, then the coefficients will be biased.

### 4.3 IPO Behavior

Our model predicts sector-specific IPO waves, occurring when  $\rho_t$  becomes high enough. Evidence supporting IPO waves is documented in Lee et al. (1991) and Cherkes (2003a), and additional support can be readily gleaned from Table 1 and Panel b. of Table 2. While a sentiment model would also predict waves of IPOs, these would occur in all sectors simultaneously, unless one is willing to admit the possibility of sector-specific sentiment.

	<b>Muni</b>	<b>Taxable FI</b>	<b>Dom. Equity</b>	<b>For. Equity</b>	<b>Other</b>
<b>term</b>	1.274 (8.54)	0.419 (8.05)	0.269 (5.19)	-0.078 (-1.12)	0.351 (9.25)
<b>corpspread</b>		-0.335 (-1.82)			-0.459 (-3.65)
<b>vix</b>	0.000 (-0.02)	0.000 (0.03)	-0.077 (-5.18)	-0.066 (-4.13)	0.052 (6.41)
<b>top_sent</b>	-0.029 (-1.82)	0.001 (0.21)	0.031 (4.66)	-0.03 (-4.11)	0.002 (0.42)
<b>variablecomponent</b>			3.695 (0.22)	-27.814 (-1.31)	29.048 (2.74)
<b>liq_level2</b>			-1.244 (-1.17)	0.463 (0.32)	-0.262 (-0.4)
<b>taxDiff</b>	-0.22 (-7.65)				
<b>Cons</b>	1.095 (0.6)	-0.524 (-0.84)	-2.505 (-3.46)	4.343 (4.92)	-1.686 (-3.83)
<b>Adj R2</b>	0.2281	0.1944	0.2686	0.1366	0.3672

Table 9: **Determinants of CEF IPOs.** Tobit regressions of the equally weighted six-month moving average of the number of IPOs each month against explanatory variables.

As with the determinants of the premium above, we run a Tobit regression of the number of funds that IPO in each year from 1986–2004 against the variables that the two models suggest ought to explain the CEF premium. Table 9 shows that the results are weaker than those of the three-stage regressions presented earlier. There is a little more evidence for the sentiment story, although it is only conclusive for both sentiment measures in the case of domestic equity funds. Moreover, there is somewhat less evidence in favor of the liquidity story. The municipal bond *taxDiff* and *term* still lend strong support to a liquidity-based model.<sup>34</sup>

### 4.4 Post-IPO Returns

In our model, investors always earn a fair expected return, despite expecting the fund to fall to a discount. In particular, there should be no difference in returns between new and seasoned funds managing very similar assets. In the sentiment model of Lee et al. (1991), new investors in the CEF earn below-market returns. Weiss (1989) found evidence of a negative post-IPO risk-adjusted return for CEFs, but she used only a small sample, and measured

<sup>34</sup>Including the sector average premium in the regression adds relatively little to the regression’s explanatory power (i.e., much of the time series effects of the sector premium are captured by the systematic variables).

returns relative to a market-wide index that might not reflect the actual holdings of CEFs. Table 10 investigates the same question using a larger sample and with a more appropriate index, reporting monthly excess returns to a strategy that is long seasoned funds (over one year old) and short unseasoned funds (less than one year old).<sup>35</sup> We examine the effects of both equal and value weighting in the portfolio construction. We also examine the effect of unlevering the returns for funds that are levered.<sup>36</sup> Once we adjust the returns at the fund level for leverage, evidence for underperformance of unseasoned funds only exists in two sectors: Muni and taxable fixed income. In particular, there is no evidence of underperformance in the much-studied domestic equity sector. Moreover, although statistically insignificant, there is evidence for economically significant *overperformance* of foreign equity unseasoned funds. The results are similar, though somewhat weaker, when unseasoned funds are defined to be two years or less from their IPO.

Sector		Value Weighted		Equal Weighted	
		Raw	Unlevered	Raw	Unlevered
<b>Muni</b>	Monthly excess return	0.0042	0.0032	0.0051	0.0034
	t-stat.	4.44	3.73	5.17	4.01
	N	185	117	185	117
<b>Taxable FI</b>	Monthly excess return	0.0035	0.0032	0.0047	0.0038
	t-stat.	2.41	2.09	3.28	2.55
	N	204	123	204	123
<b>Dom. Equity</b>	Monthly excess return	0.012	0.0073	0.0138	0.0117
	t-stat.	2.63	0.98	3.04	1.54
	N	141	64	141	64
<b>For. Equity</b>	Monthly excess return	-0.0022	-0.0171	0.0057	-0.0121
	t-stat.	-0.38	-1.6	1	-1.13
	N	160	56	160	56
<b>Other</b>	Monthly excess return	-0.0009	-0.0007	-0.0006	0.0001
	t-stat.	-0.37	-0.27	-0.28	0.07
	N	150	78	150	78

Table 10: **Returns on new vs. seasoned CEFs.** This table reports average monthly difference in returns between seasoned (over one year old) and unseasoned funds in our database.

<sup>35</sup>Details on how the age of a fund is calculated are provided in Appendix A. Each month and for each sector, we form a portfolio that contains only CEFs whose age exceeds 12 months, and another portfolio that contains only CEFs whose age does not exceed 12 months. Returns in the IPO month are excluded by construction.

<sup>36</sup>We assign a fund its average sector leverage in each reporting period that it is missing leverage data.

## 4.5 CEF vs. NAV returns

In our model, NAV returns are higher than CEF returns. The sentiment theory of Lee et al. (1991) points out that, if sentiment has a systematic component, rational arbitrageurs will demand a risk-premium, thereby causing CEFs, on average, to have higher expected returns than the NAV.<sup>37</sup> Sias et al. (2001) find that CEFs do not earn higher returns, on average, than their underlying assets. This is corroborated by Table 2 in Wermers et al. (2005). Panel a. of table 11 also examines this, analyzing NAV vs. stock returns for CEFs in our data set.<sup>38</sup> As predicted by the liquidity theory, mean NAV returns exceed mean CEF returns by a sizeable margin for four of the five fund types, though with the exception of “Other” funds, none of the differences is statistically significant.<sup>39</sup>

The models also make different predictions for *absolute* returns around the time of the fund’s IPO. In the liquidity theory, CEF shares should always earn a fair rate of return, while NAV returns should be high at times of fund inception. In the sentiment theory, the NAV should always earn a fair rate of return, but the CEF shares should earn an abnormally low rate of return immediately following an IPO. Panels b.–d. of Table 11 explore this by comparing the raw mean returns with IPO weighted returns (the average return for each year, weighted by the number of IPOs in that year, then added and divided by the total number of IPOs).

Panel b. looks at CEF stock returns from 1986–2004, while panel c. only considers the period 1994–2004, in order to better compare with the NAV return results in Panel d. For muni, taxable fixed income and other funds, raw and IPO-weighted returns are very similar, as would be expected under the liquidity theory. The sentiment theory receives some support from the domestic and foreign equity CEF sectors over the period 1986–1993, but not during 1994–2004. Panel d. of the table looks at the NAV returns available to us. It can be seen that IPO-weighted NAV returns substantially exceed raw returns for all sectors but one (even though the differences are not statistically significant).

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<sup>37</sup>Without this systematic component, the sentiment model does not make an unambiguous prediction for the unconditional average return difference.

<sup>38</sup>Because we only have expense data from 1994 on, we are restricted to looking at NAV returns only during that period. Consistent with Wermers et al. (2005), NAV returns are calculated as  $\text{navRet}_t = \ln((\text{NAV}_t + \text{dist}_t)/(1 - \text{exratio}_t)) - \ln \text{NAV}_{t-1}$ . By contrast, we have return data on our CEFs over the entire sample period.

<sup>39</sup>The fund-weighted mean is calculated by multiplying the difference in returns by the number of funds for which we have data in that year.

		<b>Muni</b>	<b>Taxable FI</b>	<b>Dom. Equity</b>	<b>For. Equity</b>	<b>Other</b>
<i>a. Difference between NAV and CEF annual returns</i>						
Raw	mean	1.8%	1.8%	2.7%	-1.1%	2.8%
	t-stat	1.37	1.40	1.40	-0.23	1.94
Fund weighted	mean	1.3%	1.4%	3.0%	-5.6%	5.0%
	t-stat	0.85	0.99	1.46	-1.08	2.79
<i>b. Mean CEF stock returns (1986-2004)</i>						
Raw		4.6%	6.7%	10.9%	13.7%	7.0%
IPO weighted		3.7%	6.1%	6.8%	4.0%	7.2%
Difference	mean	0.9%	0.6%	4.1%	9.7%	-0.2%
	t-stat	1.42	1.06	1.68	3.93	-0.18
<i>c. Mean CEF stock returns (1994-2004)</i>						
Raw		5.4%	6.5%	10.8%	9.1%	6.8%
IPO weighted		3.1%	7.3%	10.3%	-6.6%	8.0%
Difference	mean	2.3%	-0.8%	0.5%	15.8%	-1.2%
	t-stat	1.46	-0.70	0.12	1.30	-0.63
<i>d. Mean NAV returns (1994-2004)</i>						
Raw		7.3%	7.9%	12.7%	8.3%	9.3%
IPO weighted		7.3%	9.2%	17.7%	21.6%	13.6%
Difference	mean	0.0%	-1.2%	-5.0%	-13.3%	-4.2%
	t-stat	-0.01	-1.22	-1.39	-1.40	-1.90

Table 11: **NAV vs. stock returns.** This table documents CEF stock returns versus NAV returns from 1986–2004, using data described in Appendix A.

Overall, though the evidence is not overwhelming (probably due to the well-known difficulty inherent in performing direct statistical comparisons of expected returns over short periods (see Merton, 1980)), it provides some support for the predictions of the liquidity theory. NAV returns are higher than stock returns, stock returns (except equity funds) are roughly the same around IPO times as at other times, and NAV returns are higher at the time of the IPO.

## 5 Conclusions

This paper develops a rational, liquidity-based model of closed-end funds (CEFs) that provides a simple economic explanation for their existence: Since investors can sell their CEF shares without the underlying assets changing hands, there are cost savings to buying illiquid assets indirectly, via a CEF, rather than directly (or via an open-end fund). In our model, a CEF may trade at either a discount or a premium, depending on the size of the manager’s fees relative to the liquidity benefits of the fund, and the model explains CEF IPO patterns and the behavior of the premium. Analysis of a comprehensive CEF data set from

1986–2005 provides support for both the underlying economic assumptions of the model and its predictions for IPO, premium, fund return, and NAV return behavior. Moreover, our analytical model can be calibrated to fit sector-specific CEF premium behavior, including the quick reversion from a premium at IPO to a discount.<sup>40</sup>

The evidence documented in this paper also suggests that, overall, the data do not support the predictions of a sentiment model. It appears that, if there is a puzzle associated with closed-end funds, it has to do with the under-performance of certain young funds (less than 12 months from their IPO). The fact that the IPO cost for a fund is typically higher than the prevailing premium in the fund’s sector, as well as the high volatility of liquidity premium required to calibrate the model to the data, appear to indicate that some CEFs may be overpriced when they IPO. It is worth documenting that the average IPO cost in our panel has decreased from about 7% to about 4.5%. Thus it may very well be the case, going forward, that the underperformance phenomenon will disappear.

If one suspects irrational and systematic overpricing of CEF IPOs (i.e., the possibility of issuers deliberately overpricing to take advantage of unsophisticated investors), then our model provides some guidance for policy. In particular, policy makers might consider that any regulations should be aimed at preserving the valuable liquidity services that CEFs provide small investors.

Finally, our explanation for the CEF discount is applicable to any situation in which bundling securities provides liquidity benefits to investors, such as REITs, ADRs and asset-backed securities. It also provides a potential explanation for the existence of conglomerates.

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<sup>40</sup>Our data set indicates that reversion to a discount takes an average of one year, significantly longer than the 120 days noted by Weiss (1989) in her much smaller data set.

## A Data Description

Our data are collected from a variety of sources. Not every data item is available for each fund-date combination.

**CEF-level data** From Bloomberg, we obtain monthly premium and NAV data on a survivorship bias-free sample of CEFs between January, 1986 and April, 2006.<sup>41</sup> Monthly data on returns, prices, number of shares outstanding, and cash dividend distributions are obtained from CRSP and matched to funds' ticker symbols. A CRSP stock is a CEF only if the second digit in the symbol's share code (*shrcd*) is a four or a five. Dividends are determined to be in cash if the first digit in their CRSP distribution code (*distcd*) is one and the second digit is less than five. We obtain quarterly SG&A and total assets data from funds' income statements and balance sheet, available through S&P Capital IQ (CEFs typically report their management fees under SG&A). We also compare these quarterly expenses with annual management fees available on a subsample of funds through Morningstar and generally find negligible discrepancy. The S&P Capital IQ data is only available from 1993 and on. We collect fund inception dates from SDC Platinum data (Thomson Financial) and from COMPUSTAT. The former is also our source for IPO costs. The fund prospectus objective is obtained from Morningstar and supplemented with descriptions from Lipper. Finally, we calculate 1993–2004 daily TAQ trades on a subset of funds.

**Non-CEF data** The sources for the non-CEF variables used in the study are summarized in Table 6. Summary statistics for monthly trading costs documented in Table 2 are calculated by dividing the annual level data (*cmdmlevel*) provided by Joel Hasbrouck by the monthly CRSP price. These trading costs compare well with estimates of TAQ bid-ask spreads calculated for a subsample of funds. The *cmdm* variable used in the three-stage regressions is calculated by dividing a fund's *cmdmlevel* by the average price of the fund for the year. Thus *cmdm* only varies annually. This is done so as to avoid a spurious regression relation between an individual fund's premium (which has price in the numerator) and the monthly trading costs.

**Calculated variables** A fund's payout ratio is calculated by dividing the monthly cash dividend by the sum of the cash dividend and the fund's NAV. A fund's quarterly expense ratio is calculate by dividing its SG&A by the total NAV (total NAV is NAV per share times the number of shares outstanding). A fund's quarterly leverage ratio is calculated by dividing

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<sup>41</sup> We compare our premium and NAV data with a more limited sample available through COMPUSTAT and find that the average discrepancy is negligible but the standard deviation of the discrepancy is 3%.

the difference between total assets and total NAV by the total assets. For each fund, the quarterly expense ratio and leverage are interpolated to create a monthly time series. The IPO month for each fund is calculated to be the earlier date documented by Morningstar or COMPUSTAT (if a date is available from both sources). When analyzing the number of IPOs per year or per month (see Table 1, and the IPO Tobit regressions), we count the first month of trade on CRSP as the IPO month for funds without IPO data. On the other hand, when calculating the IPO month for the purpose of deducing the median fund age or the average time to discount, we only use funds for which Morningstar or COMPUSTAT IPO date data is available. A fund’s age is calculated at each date based on the IPO month. The time to discount is determined by calculating the first month after the IPO month in which the premium is negative *and* is either also negative the following month or, if no data is available the following month, is below  $-2\%$ . This is done to avoid noise in the calculation (see footnote 41). Moreover, if a fund never exhibits a discount, then the time to discount is taken to be the size of its time-series.

To obtain the IPO-weighted premium in Table 2, we first calculate the average premium in each sector every six months starting January–June, 1986. The IPO-weighted premium is the average of these values using weights equal to the number of funds times the number of IPOs in that sector in the following six months. The *excess premium* during IPOs is the IPO-weighted premium minus the average premium calculated using only the number of funds as weights. The alternative measure, “#IPOs > 4 screen”, is calculated by replacing the number of IPOs in the previous calculation with 1 if the number of IPOs is greater than four, and with zero otherwise.

## B Proof of Theorem 1

If  $\rho_t \in (0, \bar{\rho})$ , then regardless of the value of  $x_t$ , there is only a single effective state variable – specifically,  $\rho_t$ . The differential equation satisfied by  $\hat{V}(\rho_t) \equiv \frac{\text{NAV}_t}{C_t}$  when  $\rho_t = \rho \in (0, \bar{\rho})$  is given by

$$0 = \frac{\sigma^2 \rho^2}{2} \hat{V}_{\rho\rho} + \mu \rho \hat{V}_{\rho} - (\rho + r - g) \hat{V} + 1. \quad (15)$$

The homogeneous solution to this differential equation is

$$\alpha U_+(\rho) + \beta U_-(\rho),$$

where

$$U_+(\rho) = \rho^{\frac{1}{2} - \frac{\mu}{\sigma^2}} I\left(\sqrt{\left(1 - \frac{2\mu}{\sigma^2}\right)^2 + 8\frac{r-g}{\sigma^2}}, \sqrt{\frac{8\rho}{\sigma^2}}\right),$$

and

$$U_-(\rho) = \rho^{\frac{1}{2} - \frac{\mu}{\sigma^2}} K\left(\sqrt{\left(1 - \frac{2\mu}{\sigma^2}\right)^2 + 8\frac{r-g}{\sigma^2}}, \sqrt{\frac{8\rho}{\sigma^2}}\right).$$

$I(\nu, y)$  and  $K(\nu, y)$  are the modified Bessel functions of the first and second kind, respectively.  $U_+$  increases, while  $U_-$  decreases, in its argument. Moreover,  $U_-$  is singular at the origin. The constants  $\alpha$  and  $\beta$  are determined by the boundary conditions on the problem. The fact that the Wronskian,  $W(K(\nu, y), I(\nu, y)) = \frac{1}{y}$ , can be used to show that the Green's Function associated with the homogeneous differential equation is

$$G(\rho, \rho') = \begin{cases} \frac{4}{\sigma^2} U_+(\rho) \rho'^{\frac{2\mu}{\sigma^2} - 2} U_-(\rho') & \rho \leq \rho', \\ \frac{4}{\sigma^2} U_-(\rho) \rho'^{\frac{2\mu}{\sigma^2} - 2} U_+(\rho') & \rho \geq \rho'. \end{cases}$$

A particular solution to the differential equation for  $\hat{V}(\rho)$  is therefore,  $\int_0^{\bar{\rho}} G(\rho, \rho') d\rho'$ :

$$\frac{4}{\sigma^2} U_+(\rho) \int_{\rho}^{\bar{\rho}} \rho'^{\frac{2\mu}{\sigma^2} - 2} U_-(\rho') d\rho' + \frac{4}{\sigma^2} U_-(\rho) \int_0^{\rho} \rho'^{\frac{2\mu}{\sigma^2} - 2} U_+(\rho') d\rho'.$$

One now adds a solution to the homogeneous differential equation which makes the sum satisfy appropriate boundary conditions. In other words the general solution in  $\rho \in (0, \bar{\rho})$  is

$$\hat{V}(\rho) = \frac{4}{\sigma^2} U_+(\rho) \left( \alpha + \int_{\rho}^{\bar{\rho}} \rho'^{\frac{2\mu}{\sigma^2} - 2} U_-(\rho') d\rho' \right) + \frac{4}{\sigma^2} U_-(\rho) \left( \beta + \int_0^{\rho} \rho'^{\frac{2\mu}{\sigma^2} - 2} U_+(\rho') d\rho' \right).$$

The following reflecting boundary conditions must be imposed in order to 'paste' the solutions in the regions together (see Dumas (1991)):  $\hat{V}_{\rho}(0) = \hat{V}_{\rho}(\bar{\rho}) = 0$ . Implementing these conditions gives the following:

$$\beta = 0, \quad \alpha = -\frac{U'_-(\bar{\rho})}{U'_+(\bar{\rho})} \int_0^{\bar{\rho}} \rho'^{\frac{2\mu}{\sigma^2} - 2} U_+(\rho') d\rho'.$$

Substituting this, one gets

$$\begin{aligned} \hat{V}(\rho_t) &= \frac{4}{\sigma^2} U_+(\rho_t) \left( \int_{\rho_t}^{\bar{\rho}} \rho'^{\frac{2\mu}{\sigma^2} - 2} U_-(\rho') d\rho' - \frac{U'_-(\bar{\rho})}{U'_+(\bar{\rho})} \int_0^{\bar{\rho}} \rho'^{\frac{2\mu}{\sigma^2} - 2} U_+(\rho') d\rho' \right) \\ &\quad + \frac{4}{\sigma^2} U_-(\rho_t) \int_0^{\rho_t} \rho'^{\frac{2\mu}{\sigma^2} - 2} U_+(\rho') d\rho'. \end{aligned}$$

## B.1 Limit when $\rho_t$ is constant

A special case of interest is when  $\rho_t = \rho$ , a constant, i.e, when  $\mu = 0$  and  $\sigma = 0$ . Setting  $\mu = \sigma = 0$ , Equation (15) becomes

$$0 = -(\rho + r - g)\hat{V} + 1,$$

with solution

$$\hat{V} = \frac{1}{\rho + r - g}.$$

It is relatively straightforward to verify that this is also the limit of the general solution in Theorem 1, if we set  $\mu = 0$  and let  $\sigma \rightarrow 0$ . Define:

$$\begin{aligned}\nu &\equiv \frac{1}{\sigma} \sqrt{\sigma^2 + 8(r - g)}, \\ z(\rho) &\equiv \sqrt{\frac{\rho}{\sigma^2/8 + (r - g)}}, \\ \eta(z) &\equiv \sqrt{1 + z^2} + \ln \frac{z}{1 + \sqrt{1 + z^2}}.\end{aligned}$$

In what follows, the arguments of  $z$  and  $\eta$  will be suppressed, unless it is important not to do so for the sake of clarity. First, make use of the asymptotic identities 9.7.7–9.7.8 for modified Bessel Functions in Abramowitz and Stegun (1964) to write, for  $\nu$  large and  $\mu = 0$ :

$$\begin{aligned}U_+(\rho) &\sim \sqrt{\frac{\rho}{2\pi\nu}} \frac{e^{\nu\eta}}{(1 + z^2)^{\frac{1}{4}}} \left(1 + O(\nu^{-1})\right), \\ U_-(\rho) &\sim \sqrt{\frac{\pi\rho}{2\nu}} \frac{e^{-\nu\eta}}{(1 + z^2)^{\frac{1}{4}}} \left(1 + O(\nu^{-1})\right),\end{aligned}$$

where  $O(\nu^{-1})$  approaches zero at the rate  $1/\nu$ . The derivatives of these expressions, for large  $\nu$ , can be written:

$$\begin{aligned}U'_+(\rho) &\sim \sqrt{\frac{\rho\nu}{2\pi}} e^{\nu\eta} A(\rho) \left(1 + O(\nu^{-1})\right), \\ U'_-(\rho) &\sim -\sqrt{\frac{\pi\rho\nu}{2}} e^{-\nu\eta} A(\rho) \left(1 + O(\nu^{-1})\right).\end{aligned}$$

The indefinite integral of  $\rho^{-2}U_{\pm}(\rho)$  can likewise be expressed as:

$$\begin{aligned}\int \rho^{-2}U_{+}(\rho)d\rho &\sim \sqrt{\frac{1}{2\pi\nu^3\rho^3}}\frac{e^{\nu\eta}}{(1+z^2)^{\frac{1}{4}}}\frac{1}{\frac{d\eta}{dz}\frac{dz}{d\rho}}\left(1+O(\nu^{-1})\right), \\ \int \rho^{-2}U_{-}(\rho)d\rho &\sim -\sqrt{\frac{\pi}{2\nu^3\rho^3}}\frac{e^{-\nu\eta}}{(1+z^2)^{\frac{1}{4}}}\frac{1}{\frac{d\eta}{dz}\frac{dz}{d\rho}}\left(1+O(\nu^{-1})\right),\end{aligned}$$

where it can be shown, after some manipulation, that  $\frac{1}{\frac{d\eta}{dz}\frac{dz}{d\rho}} = \frac{2\rho}{\sqrt{1+z^2}}$ . Finally, it is useful to observe that, for any finite  $\nu$ ,  $\lim_{z\rightarrow 0} e^{\nu\eta(z)} = 0$ . Using all of this in Equation (6), one arrives, after some cancellation, at:

$$\hat{V}(\rho) \sim \frac{4}{\sigma^2\nu^2\rho}\frac{1}{\sqrt{1+z^2}}\frac{1}{\frac{d\eta}{dz}\frac{dz}{d\rho}} = \frac{8}{\sigma^2\nu^2}\frac{1}{1+z^2} \xrightarrow{\sigma\rightarrow 0} \frac{1}{r-g+\rho}.$$

## C Proof of Theorem 2

First note that if  $K \geq k$  then no CEF ever liquidates. Thus one only needs to worry about the entry of CEFs. Entry can only happen if the value of a managed fund exceeds  $1+u$  times the value of its underlying assets. To show (8) describes an equilibrium process with  $\bar{\rho}$  described in the Theorem, consider that (8) describes a reflected Brownian motion in  $[0, \bar{\rho}]$ . Taking this process as given, Theorem 1 gives the value of the underlying asset.  $\hat{V}(\rho_t)$  is monotonically decreasing, and  $\hat{V}(0) = \frac{1}{r-g}$ . Moreover, from the asymptotic expansion of the Bessel functions and their integral,  $\hat{V}(\bar{\rho}) \rightarrow 0$  as  $\bar{\rho} \rightarrow \infty$ . Thus the equation

$$\frac{1-k}{r-g} = \hat{V}(\bar{\rho})(1+u)$$

has a unique solution if at least one of  $k$  and  $u$  is strictly positive.

Thus, if all stake holders take the process  $\rho_t$  as given, then CEF entry takes place only at  $\rho_t = \bar{\rho}$ . Moreover, at this value, firms are indifferent between entering and not entering. If all CEFs enter at  $\rho_t = \bar{\rho}$  then  $x_{t+} = 1$  which is inconsistent with the posited process. On the other hand, if no CEF enters, then  $\rho_t = \rho_t^f q(x_t)$  does not get reflected at  $\rho_t = \bar{\rho}$ . Thus a consistent equilibrium strategy must be mixed. To derive the equilibrium strategy, consider that  $dx_t = \frac{1}{q'(x_t)}d\frac{\bar{\rho}}{s_t^f}$  is the increase in the amount of illiquid asset under CEF management. Given that there are  $1-x_t$  units of CEFs that could potentially enter, if each unit enters with probability  $\bar{\rho}\frac{1}{1-x_t}\frac{1}{q'(x_t)}d\frac{1}{s_t^f}$ , then the total amount of entry is  $\bar{\rho}\frac{1}{q'(x_t)}d\frac{1}{s_t^f} = dx_t$ , as required. Summarizing:  $\rho_t$  is an equilibrium supported by a mixed-strategy entry policy.  $\square$

In the text we remark that, in view of Assumptions 8 & 5, the proportion of illiquid asset

under CEF management can never decrease with time. We also remark that the model can be readily re-interpreted so as to do away with this peculiar feature. To see this, define

$$Q_t = \bar{\rho} \frac{y_t}{\max\{\bar{\rho}, \sup_{\tau \leq t} \rho_t^f y_t\}},$$

where  $y_t$  is geometric Brownian motion (as is  $\rho_t^f$ ) and  $\frac{d(\rho_t^f y_t)}{\rho_t^f y_t} = \mu dt + \sigma dW_t$ . Then our results are the same if one writes  $\rho_t = \rho_t^f Q_t$ . Here,  $Q_t$  can be viewed as a monotonically increasing transform of the supply of the asset. Moreover, our original assumptions can be recovered by setting  $y_t = 1$  for all  $t$ . Under this re-definition, the supply of illiquid asset no longer weakly decreases with time.

## D Proof of Theorem 3

The probability density function for  $\rho_t$ , if it is stationary, can be derived from the Fokker-Planck equation:

$$\frac{\partial^2}{\partial \rho^2} \left( \frac{\sigma^2 \rho^2}{2} f(\rho) \right) - \frac{\partial}{\partial \rho} \left( \mu \rho f(\rho) \right) = 0.$$

The solution to this equation is  $f(\rho) = \frac{A}{\rho} + B \rho^{2(\mu - \sigma^2)/\sigma^2}$ . Since the cumulative distribution should vanish at  $\rho = 0$  for a stationary process (i.e., recall 0 is an absorbing barrier), stationarity requires  $2\mu > \sigma^2$ . In addition, the integral of  $f(\rho)$  between 0 and  $\bar{\rho}$  is unity, so the solution is

$$f(\rho) = \frac{\gamma}{\rho} \left( \frac{\rho}{\bar{\rho}} \right)^\gamma.$$

where  $\gamma = 2\mu/\sigma^2 - 1$ . The cumulative distribution function,  $F(\rho)$ , follows from integrating  $f(\rho)$ .

To calculate the expected time to reversion from an IPO, consider the pricing of a perpetual barrier option that pays \$1 when the reflected process  $\rho_t$  falls below the level  $\rho_0$ , assuming the interest rate is  $\alpha$ . The value of such a security can be written as  $W(\rho_t) = E_t[e^{-\alpha\tau}]$  where  $\tau = \inf_{t \leq t' \leq \infty} \{t' \mid \rho_{t'} \leq \rho_0\}$  is a stochastic hitting time. The expected time that it takes for  $\rho_t$  to get to  $\rho_0$  is defined as  $T(\rho_t, \rho_0) \equiv -\frac{\partial W(\rho_t)}{\partial \alpha} \Big|_{\alpha=0}$ . To find this expression, we note that for  $\rho_0 < x < \bar{\rho}$ ,  $W(x)$  is a solution to the equation

$$\frac{\sigma^2 x^2}{2} \frac{\partial^2 W}{\partial x^2} + \mu x \frac{\partial W}{\partial x} - \alpha W = 0,$$

with the value matching boundary condition  $W(\rho_0) = 1$  and the reflecting barrier condition  $W'(\bar{\rho}) = 0$ . The general solution is  $W(x) = A_+ x^{\gamma_+} + A_- x^{\gamma_-}$ , where  $\gamma_{\pm} = (-\gamma \pm$

$\sqrt{\gamma^2 + 8\alpha/\sigma^2})/2$ . Note that for  $\alpha > 0$ ,  $\gamma_- < 0 < \gamma_+$ . Imposing the boundary conditions and solving for  $A_{\pm}$  gives

$$W(\rho_t) = \frac{\gamma_+ \left(\frac{\rho_t}{\bar{\rho}}\right)^{\gamma_-} - \gamma_- \left(\frac{\rho_t}{\bar{\rho}}\right)^{\gamma_+}}{\gamma_+ \left(\frac{\rho_0}{\bar{\rho}}\right)^{\gamma_-} - \gamma_- \left(\frac{\rho_0}{\bar{\rho}}\right)^{\gamma_+}}.$$

To calculate the hitting time from  $\rho_t = \bar{\rho}$ , we differentiate  $W(\rho_t)$  with respect to  $\alpha$  and set  $\alpha = 0$  and  $\rho_t = \bar{\rho}$ . After some manipulation one arrives at our expression for  $T(\rho_0) = T(\bar{\rho}, \rho_0)$ .

To work out the distribution of  $\rho_t$  a time  $t$  after the IPO, note that the joint distribution density for the maximum and level ( $m$  and  $b$ , respectively) of a Brownian Motion process with drift  $\theta$  is given by

$$\frac{2(2m - b)}{\sqrt{2\pi t^3}} \exp\left(-\frac{(2m - b)^2}{2t} + \theta b - \frac{1}{2}\theta^2 t\right),$$

where, initially,  $m_0 = 0 = b_0$ , and  $m > 0, b < m$ .<sup>42</sup> Letting  $\theta \equiv \frac{\mu}{\sigma} - \frac{\sigma}{2}$ , the liquidity premium  $\rho_t$  can be written as  $\rho_t = \bar{\rho}e^{\sigma(b_t - m_t)}$  (assuming the IPO took place at  $t = 0$ ). Thus the expectation over any function of  $\rho_t$  given  $t = 0$  information can be calculated by integrating over the distribution function. The expression in the theorem is calculated by making the change of variables  $u = \frac{m+b}{\sqrt{2}}$  and  $v = \frac{m-b}{\sqrt{2}}$ , and computing the integral over  $u$ .  $\square$

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<sup>42</sup>See Theorem 7.2.1 in Shreve (2004).

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