Energy Efficiency and Commercial-Mortgage Valuation*

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Abstract

Energy efficiency is a key factor to the future of the U.S. economy, and commercial buildings are among the largest users of energy. However, existing commercial mortgage underwriting practices provide little incentive for building owners to make their buildings more energy efficient. In this paper, we extend standard mortgage valuation methods, which account for the expected dynamics of interest rates and building prices, by including the expected dynamics of the electricity and gas forward prices as well as average building-level energy consumption. This allows us explicitly to incorporate energy risk and efficiency measures, which depend on both the energy efficiency of the building and the characteristics of its location, into commercial mortgage valuation and underwriting. We apply our valuation methodology to price a sample of 1,390 mortgages, originated between 2005 and 2007 on office buildings located in 28 cities across the U.S. We find that, relative to the traditional mortgage valuation methodology, our proposed strategy leads to an 5% reduction in the mispricing of the default risk of commercial mortgages.

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1 Introduction

Real estate structures account for almost 40 percent of total U.S. energy consumption, with commercial real estate alone consuming 18 percent of the total (see Table 1). The result is that energy costs represent a substantial share of the total operating costs for U.S. commercial buildings (for example, 31 percent for U.S. office buildings). U.S. real estate is also, on average, far less energy efficient than comparable European buildings after controlling for building type, size, and climate.[1] U.S commercial property owners simply have not applied the available technology for improving energy efficiency. The historically low prices for U.S. energy were, of course, a major reason for their inaction. But as energy prices are now rising and becoming more volatile, and as energy markets become globally integrated, this is rapidly changing. There is, indeed, improvement in the energy efficiency of new U.S. construction, but for existing commercial buildings the retrofit rate remains remarkably slow. At this pace, it will be decades before the U.S. stock of commercial real estate reaches an economically reasonable level of energy efficiency.[2]

Resid. Bldgs	Comm. Bldgs	Total Bldgs	Industry	Transport	Total
20.9%	18.0%	38.9%	32.7%	28.4%	100%

Table 1: Buildings Share of U.S. Primary Energy Consumption (Percent), 2006.
Source: U.S. Department of Energy (2009).

Public policy has responded to the energy inefficiency of U.S. real estate in several ways. First, at the local and state levels, building codes and similar construction restrictions now require higher levels of energy efficiency. However, these rules apply primarily to new construction, and they may even have the negative effect of discouraging innovations and their adoption before they reach the codes. Second, energy efficiency disclosure certificates are now available, both from government agencies (e.g., the Department of Energy's *Energy Star* Program) and from the private sector (e.g., the *LEED* program from the U.S. Green Building Council). However, these programs also have their primary impact on new construction. Further, while it appears that certified buildings are generally more energy efficient, it is unclear whether their higher construction costs are adequately reflected in higher asset values. Thus, it remains unclear whether the certificates actually stimulate economically constructive energy-saving investments. Finally, there are a variety of subsidy programs available from federal and state governments. But as a result of the currently

[1] IEA (2008) provides comparisons of energy use in the U.S. and Europe corrected for climate and measured per unit of GDP or per capita. McKinsey (2007) also shows substantially higher U.S. energy consumption compared to Europe after controlling for GDP and population. Rand (2009) provides a discussion comparing energy use in the U.S., Australia, and the European Union.

[2] Similar concerns apply to the U.S. single-family housing stock, and there are broad parallels in both the sources and solutions between commercial and single-family real estate. However, the details are quite different and in this paper we focus on commercial real estate.

large macroeconomic fiscal deficits, these programs will more likely contract than expand for the foreseeable future.

An alternative, and more fundamental, approach to rectify the energy efficiency problem is to eliminate, where possible, any private market failures that continue to inhibit energy efficiency investments in real estate, even as the level and volatility of energy prices are rising. We have identified two primary failures. First, only limited information is available to identify energy efficient investments and to determine if they have been implemented in a given building. For example, while the various certificate programs were introduced for this reason, as already suggested, they do not provide precise building-level tools for analyzing and carrying out energy efficiency investments. Similarly, while most commercial mortgage lenders require a detailed engineering report on the building that serves as collateral for the loan (called the Property Condition Assessment (PCA)), currently the PCA is primarily used to determine if the building's existing systems are in need of repair, and if so to ensure that suitable reserve accounts are incorporated into the loan amount. The PCA reports are rarely, if ever, used to evaluate the energy efficiency of a building or to recommend possible improvements.[3]

Second, the current, standard underwriting methodology for commercial mortgages provides little or no incentive to implement energy-saving investments. This methodology generally sets a desired loan to value ratio (LTVR, say $\leq 65\%$) and a desired debt service coverage ratio (DSCR, say ≥ 1.25). Both ratios are based, directly or indirectly, on the amount of net operating income (NOI) generated by the property. While energy costs are, in principle, a component of NOI, the standard ARGUS software used to create the NOI pro formas for commercial lenders has no explicit energy cost input. This may make sense for properties with triple net leases, since the tenants directly pay the majority of the building's energy expenses. However, such tenants may also be willing to pay higher rents to obtain space in buildings with lower energy costs, but this link of greater energy efficiency to higher NOI is not integrated into the pro formas.

The absence of energy efficiency information for commercial mortgage lenders has the result that these lenders are unable to distinguish the relative default risk of energy efficient buildings versus inefficient buildings, and consequently they do not risk-adjust the mortgage interest rates or loan sizes on buildings with differing energy efficiency attributes. Thus, current building owners generally do not realize lower mortgage interest rates on more energy efficient buildings. For the same reason, lenders are reluctant to fund the larger loans required on buildings that embed greater energy-saving investments. Energy retrofits on existing structures are also difficult to finance due to the lack of existing underwriting methods that allow lenders to price the risk mitigation benefits of these retrofits. For lenders to accurately price energy-saving benefits, the traditional commercial mortgage valuation and underwriting strategies must be augmented to explicitly include energy-related sources of risk.

The energy information failure for commercial mortgage lenders can also be interpreted within

[3]We believe that the PCA reports have a large potential to become the vehicle through which information on the energy efficiency of a building can be transferred from engineers to the building owner and mortgage lender. This potential role of the PCA is discussed at greater length in the DOE report from which this paper is derived.

standard finance theory. If we assume that building owners have full information on energy-saving investments, and if the principal-agent issues with tenants are resolved, then an all-equity financed building should achieve the first best level of energy efficiency. Furthermore, if the conditions of the Modigliani and Miller (1958) (M&M) theorem are satisfied, then even mortgage-financed buildings should reach this first-best investment level. However, if mortgage lenders are not as well informed as the building owners, then this informational asymmetry breaks down the M&M result. Two conclusions follow: (i) Savvy building owners will save the costs of making energy-saving investments, in effect achieving a free transfer of the default risk to the poorly informed lender; (ii) If lenders are aware of this risk transfer, but cannot identify the offending buildings, they may treat all buildings as lemons, charging all owners higher mortgage rates to offset the higher default risk on the inefficient buildings.

Beyond the commercial mortgage underwriting issue, there is also the question of whether building owners would want to make energy-saving investments on buildings with triple-net leases, in order to garner higher rents from appreciative tenants. While the answer is yes in principle, three conditions must be met for such investments to occur in practice:[4]

1. Tenants must be confident they will face lower energy costs before they will pay higher rents;
2. Building owners must be confident they will receive higher rents before they will make energy-saving investments;
3. The energy-saving investments will actually be undertaken only when it is demonstrably clear that the investments have a positive net present value.

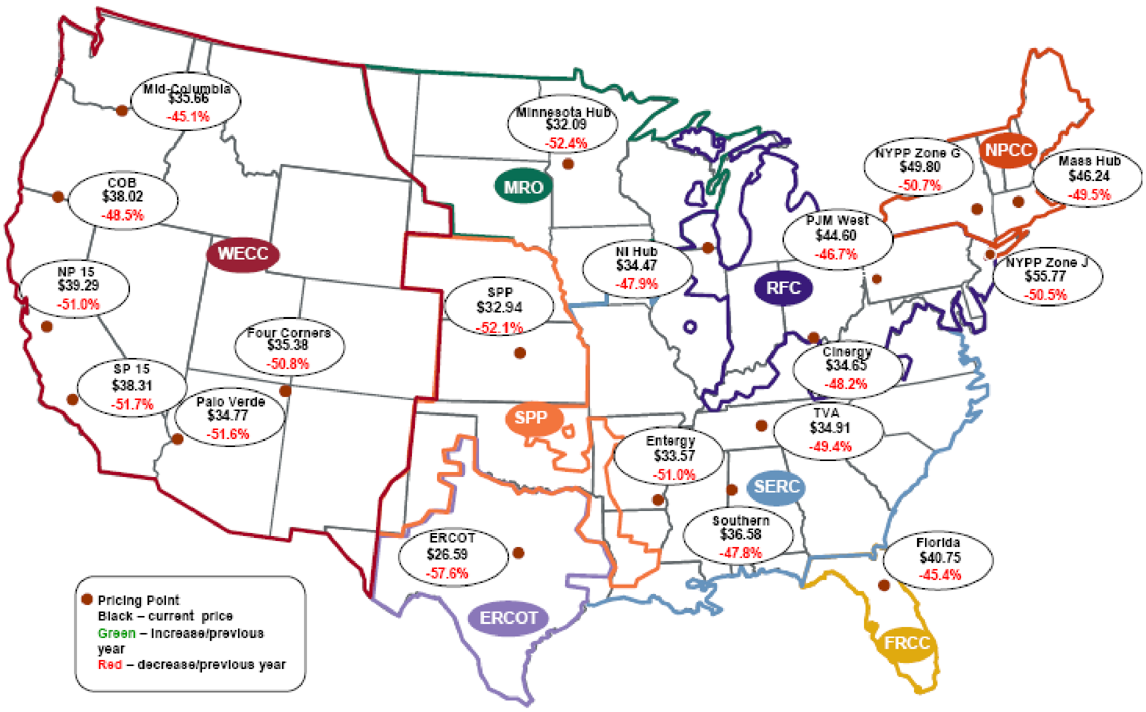
The conclusion is that lenders, tenants, and building owners alike must be able to identify energy-saving investments and objectively determine that the benefit-cost ratio is positive before they will participate in the coordinated enterprise that creates energy-efficient buildings. To date, the lack of information has frustrated actions, creating in effect a coordination problem. Our recommended approach to solve this coordination problem is to develop information and valuation tools that can be applied by lenders and building owners in evaluating and then funding energy-saving investments in commercial buildings.

The strategy in this paper is to develop a commercial mortgage valuation and underwriting tool that explicitly accounts for the energy risk of individual buildings. The analysis requires a systematic dynamic dimension that simultaneously accounts for changing interest rates, property values, and energy costs. A key and necessary innovation in our methodology is to explicitly model the rent and operating cost dynamics of the building, with a new and unique focus on the expected price and quantity of the electricity and natural gas used to operate the building. The resulting model, for the first time, provides commercial property owners and mortgage lenders with a tool that explicitly measures the net benefits of energy-saving investments.

The analysis also requires a systematic regional component because (i) climatic conditions vary across regions, leading to differing benefits from energy-saving investments, and (ii) the cost of

[4]The agency problem between building owners and tenants vanishes, of course, with owner-occupied buildings. However, the lender must still approve the larger loan required on a building with energy-saving investments.

Figure 1: Federal Energy Regulatory Commission Geographic Location of the Power Hubs in the United States
This figure was obtained from the Federal Energy Regulatory Commission (`www.ferc.gov/oversight`). It presents the geographic location of the hubs for electricity forward contract auctions in the U.S.. The average dollar value of the near contract over the year 2009 is presented for each hub and the percentage change in this average price from the average over the year 2008.

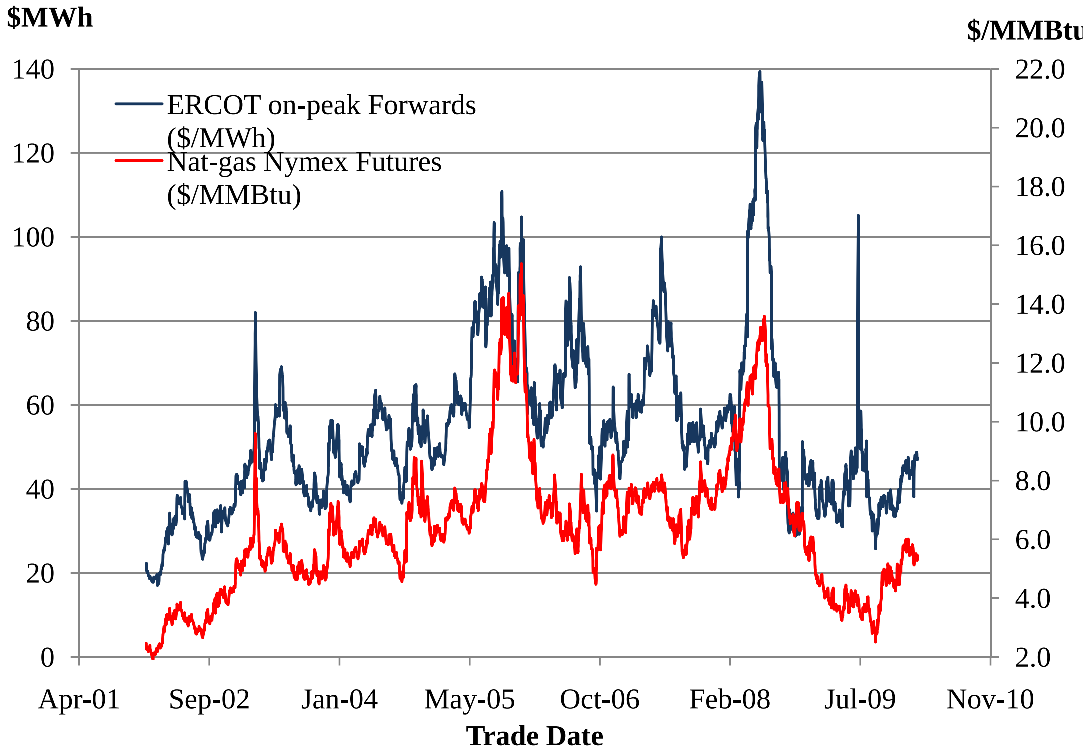


the energy used in buildings (electricity and natural gas) can vary significantly across regions. As an example of the regional variation in energy prices, Figure 1 shows average near-term forward contract prices for 2009 across the major U.S. electricity hubs. The price of both electricity and gas can also be highly volatile over time. Figure 2 plots time series for the monthly electricity forward prices for the Texas electricity hub (the ERCOT hub) and the Henry Hub natural gas futures contract.[5] It is clear from Figure 2 that there have been periods of significant differences in the dynamics of monthly futures prices for natural gas and forward prices for electricity. Overall, the electricity forward price dynamics appear more volatile and they appear to exhibit a stronger seasonal component. While both series mean revert, the speed of mean reversion of the electricity forward prices appears more rapid than that for natural gas futures.

In this study, we benchmark the existing energy efficiency of buildings by location and size classes using two sources developed by the Lawrence Berkeley National Laboratory: the Commercial

[5]The Henry Hub is at the center of the U.S. natural gas pipeline system and is the primary basis for natural gas pricing.

Figure 2: Nearest Contract Price for the ERCOT Electricity Forward Contracts and Henry Hub Natural Gas Futures Contracts



Buildings Energy Consumption Survey (CBECS),[6] and the California Commercial End-Use Survey (CEUS)[7] (see Mathew, Mills, Bourassa, and Brook, 2008a; Mathew, Mills, Bourassa, Brook, and Piette, 2008b). As shown in Table 2, there is considerable variability in the consumption levels of natural gas and electricity across regions and building types. In general, the western coastal regions appear to have lower consumption levels of both electricity and natural gas and the East Coast and Texas locations appear to have higher consumption levels. There are also important differences across buildings with different square footage. These reported median energy consumption variables, as will be discussed below, will have important implications for pricing the relative risk of mortgage across locations and building sizes. A significant limitation of the CEUS and CBECS data, which are currently the most comprehensive data sets on natural gas and electricity consumption, is that they do not include information for a number of important metropolitan areas such as Washington, DC and Las Vegas. This limitation restricts our empirical work to those cities for which we have consumption benchmark data.[8]

These regional and dynamic features of electricity and natural gas prices are important for mortgage valuation because energy costs average about 12% of base rents, and in many regions of the country are as much as 30% of total costs.[9] Even though energy markets are regulated and most buildings do not pay the wholesale prices for power and natural gas, there is an expanding trend for real estate operating companies and local governments to purchase their electricity from the wholesale market. In addition, the wholesale markets reflect the true resource costs of energy consumption and these costs are incorporated, in time, into the rate schedules offered by regulated utility companies. Clearly, the resource signals from these markets should be of concern to mortgage lenders who bear the residual default risk associated with the energy cost exposure of borrowers.

The rest of this paper is organized as follows. Section 2 provides an overview of the theoretical framework we apply to value the real estate properties and their mortgages based on NOI, energy prices, and interest rates. It also provides the empirical calibrations for the stochastic processes underlying interest rates, rents, and electricity and natural gas prices. Section 3 provides the full specification and implementation of our mortgage and property valuation techniques. Section 4 applies the valuation techniques to a sample of properties and their associated mortgages. It reports the results of simulation experiments that demonstrate the reduction in mortgage values when energy costs are properly considered in computing the mortgage values. It also reports simulation results that show the increase in mortgage value that would arise if energy-saving investments were

[6]CBECS is a national sample survey that collects information on the stock of U.S. commercial buildings, their energy-related building characteristics, and their energy consumption and expenditures (see Commercial Building Energy Consumption Survey 2003, Energy Information Administration (EIA), `http://ww.eia.gov/emeu/cbecs/contents.html`. CBECS contains 5,215 sample building records across the country which were statistically sampled and weighted to represent the entire stock of national wide commercial building.

[7]CEUS is a comprehensive study of commercial sector energy use in California, primarily designed to support the state's energy demand forecasting activities (Itron, 2006).

[8]In an actual application, the underwriter would use the buildings utility bills to measure actual historical consumption per square foot.

[9]See BOMA Experience and Exchange Report for 2009, `http://www.boma.org/resources/benchmarking/Pages/default.aspx`, and authors' calculations based on building-owner interviews.

Table 2: CEUS Benchmarks for Median Energy Consumption by Building Size and Region

Market Name	CBECS (or CEUS) Region[a]	Large Building Consumption (Greater than 150,000 Square Feet)		Medium Building Consumption (25,000 to 150,000 Square Feet)		Small Building Consumption (Less than 25,000 Square Feet)	
		Electricity (kWh/sf-yr)	Gas (kBTU/sf-yr)	Electricity (kWh/sf-yr)	Gas (kBTU/sf-yr)	Electricity (kWh/sf-yr)	Gas (kBTU/sf-yr)
Atlanta	South Atlantic	21.8	3.7	15.3	15.5	13.0	29.2
Austin	West South Central	25.0	2.6	21.8	5.3	13.7	25.0
Boston	NorthEast	18.4	31.3	7.9	34.7	10.0	44.1
Charlotte	South Atlantic	21.8	3.7	15.3	15.5	13.0	29.2
Chicago	East North Central	21.9	17.6	13.8	40.5	10.3	54.5
Cincinnati	East North Central	21.9	17.6	13.8	40.5	10.3	54.5
Cleveland	East North Central	21.9	17.6	13.8	40.5	10.3	54.5
Dallas/Ft Worth	West South Central	25.0	2.6	21.8	5.3	13.7	25.0
Detroit	East North Central	21.9	17.6	13.8	40.5	10.3	54.5
Hartford	NorthEast	18.4	31.3	7.9	34.7	10.0	44.1
Houston	West South Central	25.0	2.6	21.8	5.3	13.7	25.0
Indianapolis	East North Central	21.9	17.6	13.8	40.5	10.3	54.5
Los Angeles	South Coast (CEUS)	14.2	6.5	13.8	7.4	12.5	9.7
Miami	South Atlantic	21.8	3.7	15.3	15.5	13.0	29.2
Milwaukee/Madison	East North Central	21.9	17.6	13.8	40.5	10.3	54.5
Minneapolis/St Paul	Mid West	21.2	16.3	14.6	40.1	10.1	42.9
New York City	NorthEast	18.4	31.3	7.9	34.7	10.0	44.1
Northern New Jersey	NorthEast	18.4	31.3	7.9	34.7	10.0	44.1
Orlando	South Atlantic	21.8	3.7	15.3	15.5	13.0	29.2
Riverside (California)	South Inland (CEUS)	18.1	10.7	13.8	8.1	11.8	12.0
Sacramento	Central Valley (CEUS)	13.3	15.2	13.1	12.6	10.1	17.6
San Antonio	West South Central	25.0	2.6	21.8	5.3	13.7	25.0
San Diego	South Coast (CEUS)	14.2	6.5	13.8	7.4	12.5	9.7
San Francisco	Central Coast (CEUS)	13.8	20.5	12.0	13.4	9.9	12.2
St. Louis	Mid West	21.2	16.3	14.6	40.1	10.1	42.9

[a] These data were provided by the Lawrence Berkeley National Laboratory.

to reduce the energy consumption of the buildings. Section 5 provides conclusions.

2 Commercial Real Estate Mortgage Valuation with Energy Risk

As previously discussed, the traditional first lien mortgage valuation process focuses on the dynamics of interest rates and building prices to model the market price of the mortgage cash flows and, heretofore, has not explicitly underwritten the risk of the cost and consumption levels of energy factor inputs for buildings. To account for energy risk, building prices must be decomposed into market rents minus total costs including the costs of energy expenditures. The canonical representation for the market price of a commercial real estate asset is as the discounted present value of the asset's future net operating income. Since well-maintained office properties typically can be assumed to be long-lived assets, the market price per square foot of a commercial office building at the investor's purchase date ($t = 0$) can be written as

$$P(0) = \sum_{t=1}^{\infty} \frac{E_0\left[NOI(t)\right]}{(1+i_t)^t}, \tag{1}$$

where $P(0)$ is the market price per square foot at the investment date, $t = 0$, $E_0\left[NOI(t)\right]$ is the expected net operating income per square foot at the t^{th} period, and the discount rate for cash flows at date t, i_t, equals the riskless rate plus a risk premium. This can alternatively be written in terms of "risk-neutral" expectations (see Harrison and Kreps, 1979) as

$$P(0) = E_0^{*}\left[\sum_{i=1}^{\infty} NOI(i\,\Delta t) e^{-\Delta t \sum_{j=0}^{i-1} \tilde{r}_j\,\Delta t}\right], \tag{2}$$

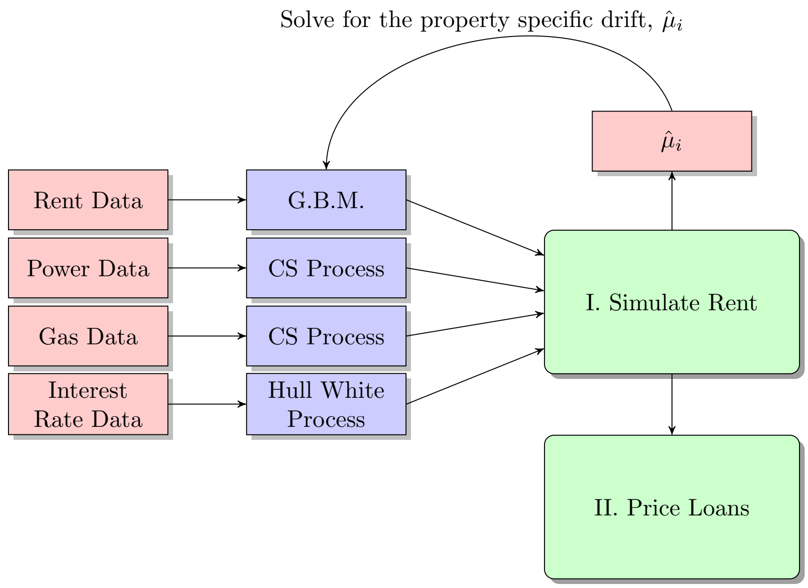
where $\tilde{r}_t$ is the one-period riskless interest rate at date t. The net operating income per square foot of an office building is defined as

$$NOI(t) = c(t) - (p_{gas}(t) \times q_{gas}(t)) - (p_{elec}(t) \times q_{elec}(t)) - (p_{other}(t) \times q_{other}(t)), \tag{3}$$

where $c(t)_{psf}$ is the rent per square foot, $(p_{gas}(t) \times q_{gas}(t))$ is the total gas expense per square foot (the price $p_{gas}(t)$ per square foot times the quantity of gas used per square foot $q_{gas}(t)$), $(p_{elec}(t) \times q_{elec}(t))$ is the total electricity expense per square foot (the price $p_{elec}(t)$ per square foot times the quantity of electricity used per square foot $q_{elec}(t)$), and other expenses per square foot, $(p_{other}(t) \times q_{other}(t))$.

The challenge of this decomposition is that the mortgage valuation problem must now account for the dynamics of four stochastic processes: 1) interest rates; 2) electricity forward prices at the appropriate geographic hub in which the property is located; 3) gas futures prices at the Henry Hub; and 4) office market rents for the building that is the collateral on the loan that is to be priced. A schematic for our proposed modeling strategy is presented in Figure 3. Moving from left to right in the figure, our mortgage valuation protocol requires market-specific data for interest

Figure 3: Flow Chart for the Mortgage Valuation Strategy



rates, electricity prices, natural gas prices, and office market rents. As previously discussed, the electricity price data is specific to the electricity hub in which the building is located. We assume that the natural-gas dynamics are determined by the NYMEX Henry Hub futures price dynamics, which are common to all office buildings in the U.S. The interest rate process, for which we use U.S. Treasury data, is also common across all buildings. The rental process must be calibrated for each building, as will be discussed in more detail below.

The data requirements for the augmented mortgage valuation protocol are also significant. Our valuation protocol requires the interest rate process, the electricity price process, and the natural gas price process to "match" (exactly fit) the observed term structure of interest rates or forward contract prices for every month for which we intend to price mortgage contracts. This requires that we collect monthly data series from 2002 through 2007 corresponding to the sample of mortgages that we will price. In addition, the natural gas and electricity simulations also require information on the expected building specific consumption levels of natural gas and electricity per square foot. As previously discussed, we use the CEUS and CBECS benchmarking values by matching buildings to locations and their appropriate building size.[10]

As shown in Figure 3, the next component of the valuation protocol is to fit a Hull-White process for interest rates and a model developed by Clewlow and Strickland (2000) (the CS Process) for fitting electricity and natural gas prices. As will be discussed below, these functional forms are commonly used in modeling these dynamics in both the practitioner and academic literatures. The price dynamics for each of the stochastic components of the model are fit exogenously using market

[10]This strategy does not allow the demand for power or natural gas to fluctuate as a function of prices. However, there is considerable evidence that office buildings in the U.S. are sufficiently inefficient that they are unable to make such price related adjustments.

data from each of the respective markets.

In *Stage I* of the modeling protocol, we solve for the implied risk-neutral drift, $\hat{\mu}_i$, of the building specific market rental process, assumed to be a Geometric Brownian motion (GBM), conditional on the estimated dynamics of the interest rates, electricity forward prices, and natural gas forward prices. The solution for this implied drift is the value that will exactly match the observed price of the building at the origination date of the mortgage given the market dynamics of the three other market fundamentals. Once the drift parameter of the building specific rent is optimally fit, the valuation component of the model, the *Stage II* component, applies the four stochastic factors: 1) interest rates; 2) electricity forward prices; 3) natural gas futures prices; and 4) the market rents for the building in a Monte Carlo simulation to compute the expected value of the contractual mortgage cash flows and the value of the embedded default option. To recap the stages of the modeling process:

1. Monthly data are assembled for U.S. interest rates; electricity forward prices by electricity hub; and natural gas forward prices for the Henry Hub;
2. The interest rate is fit to a Hull-White process and the gas and electricity price data are fit to a CS processes. These processes are fit to exactly match the observed term structure of these series on a monthly frequency.
3. *Stage I*: Using the fitted dynamics of interest rates and energy forward prices, the long run mean, or drift, of the stochastic price process for a building's market rent dynamic is fit, assuming that the process follows a GBM, such that the estimated process exactly matches the observed building price at the origination of the mortgage.
4. *Stage II*: Using a four factor model (interest rates, natural gas forward prices; electricity hub forward prices, and the building specific rental price dynamic), Monte Carlo simulation is used to value the mortgage contract cash flows and the embedded default option.

2.1 Interest Rate Dynamics

In practice for mortgage valuation, interest rate models are fit to observed market data for the term structure of interest rates and the volatility of interest rates. The Hull and White (1990) model is a commonly assumed model for this application due to its flexibility in exactly matching observed term structures and volatilities. In the Hull and White (1990) model, the short-term riskless rate is assumed to follow the risk-neutral process

$$dr_t = (\theta_t - \alpha_r r_t)\,dt + \sigma_r\,dW_t, \tag{4}$$

where dW_t defines a standard Brownian motion under the risk-neutral measure, and θ_t, α_r, σ_r and r_0 (the starting rate at time zero) are the parameters that need to be estimated. The function θ_t is fit so that the model matches the yield curve for the U.S. LIBOR swap rate on the mortgage origination date. Hull and White (1990) show that θ_t is given by

$$\theta_t = F_t(0,t) + \alpha_r F(0,t) + \frac{\sigma_r^2}{2\alpha_r}\left(1 - e^{-2\alpha_r t}\right),$$

where $F(0,t)$ is the continuously compounded forward rate at date 0 for an instantaneous loan at t. Parameters α_r and σ_r are fit with maximum likelihood.

The single-factor Hull-White model is used to simulate the interest rate process for each mortgage valuation, where the model's parameters for a given mortgage are calibrated to the LIBOR swap curve and related derivatives from the date the mortgage was issued. For each mortgage origination date these parameters were calibrated by pricing ten quarterly paying at-the-money LIBOR caps (maturities one to ten years) using discount factors and forward rates obtained from the date's LIBOR swap curve. We obtained closing quotes from Bloomberg for the 1011 trading days between 01/01/2004 and 12/31/2007 for the LIBOR deposit rates, Eurodollar futures contracts, and LIBOR spot starting swaps to construct LIBOR swap curves. We used at-the-money (ATM) LIBOR cap volatilities and strikes to calibrate the Hull-White processes.[11]

2.2 Rent Dynamics

The market rent of an office building, as discussed above is assumed to follow a geometric Brownian motion,

$$dC_t = \hat{\mu} C_t\, dt + \phi_C C_t\, dW_t, \tag{5}$$

where $\hat{\mu}$ is the risk adjusted long run drift of the rental process and ϕ_C is the volatility. The process defined by equation (5) is fit individually for each building that is the collateral for each mortgage. The results of this fitting process is will be discussed in detail below. The estimate for volatility was estimated in Stanton and Wallace (2011) to be $\phi_C = 21.478$, by solving for the implied volatility from a large sample of 9,778 office building loans originated between 2002 and 2007.

2.3 Electricity and Gas Dynamics

We calibrate the dynamics of electricity and natural gas prices following Schwartz (1997) and Clewlow and Strickland (1999), assuming that forward prices for electricity (e) and natural gas (g) follow the risk-neutral processes

$$dF_{e,g}(t,T)/F_{e,g}(t,T) = \sigma_{e,g} e^{-\alpha_{e,g}(T-t) dW_{e,g}(t)},$$

where $\sigma_{e,g}$ is the level of the spot price volatility and $\alpha_{e,g}$ is the rate of decay of the term structure of volatilities. By no-arbitrage, the spot price is equal to the forward price at t,

$$S_{e,g}(t) = F_{e,g}(t,t).$$

[11]These are Black (1976) volatilities, and thus not exactly compatible with our assumed interest rate model. The next version of this paper will convert these quoted values to implied Hull and White (1990) volatilities.

Following Clewlow and Strickland (1999), the spot prices follow a mean reverting process, with mean reversion rate equal to the rate of decay of volatility:

$$dS_{e,g}(t)/S_{e,g}(t) = [\mu_{e,g}(t) - \alpha_{e,g} \ln S_{e,g}]\; dt + \sigma_{e,g}\, dW(t). \tag{6}$$

To match the initial forward curve for electricity and futures curve for natural gas, we need to set

$$\mu_{e,g}(t) = \frac{\partial \ln F_{e,g}(0,t)}{\partial t} + \alpha_{e,g} \ln F_{e,g}(0,t) + \frac{\sigma_{e,g}^2}{4}\left(1 - e^{-2\alpha_{e,g} t}\right). \tag{7}$$

Clewlow and Strickland (1999) show that

$$\begin{aligned} F_{e,g}(t,T) = F_{e,g}(0,T) & \left(\frac{S_{e,g}(t)}{F_{e,g}(0,t)}\right)^{\exp(-\alpha_{e,g}(T-t))} \\ & \exp\left[-\frac{\sigma_{e,g}^2}{4\alpha_{e,g}} e^{-\alpha_{e,g}T}\left(e^{2\alpha_{e,g}t} - 1\right)\left(e^{-\alpha_{e,g}T} - e^{-\alpha_{e,g}t}\right)\right]. \end{aligned} \tag{8}$$

In other words, the forward (futures) curve at any future time is simply a function of the spot price at that time, the initial forward (futures) curve, and the volatility function parameters for electricity and natural gas, respectively.

Calibrating the No-Arbitrage Forward Curve We fit the continuous time no-arbitrage forward curve as a function of a secular and a seasonal factor. Following Riedhauser (2000), the secular component gives the average or trend behavior of the observed forward/futures prices on a given date and the seasonal component gives the harmonic dependence. The secular component is expressed as;

$$\Psi_{e,g}(S_{e,g}, L_{e,g}, a_{e,g}, b_{e,g}, T) = (S_{e,g} - L_{e,g}) \exp^{-b_{e,g}T} + L_{e,g} \exp^{a_{e,g}T}, \tag{9}$$

where $S_{e,g}$ is the seasonally adjusted current spot price, $L_{e,g}$ is the long term seasonally adjusted spot price, $b_{e,g}$ is the rate at which current differences from typical conditions return to the long run mean, $a_{e,g}$ is the rate of change in the (seasonally adjusted) forward price due to annual escalation of expected spot and risk adjustment, and T is time from the present. The seasonal factor includes three harmonics in the form:

$$\Omega_{e,g}(A_{e,g,1}, \tau_{e,g,1}, A_{e,g,2}, \tau_{e,g,2}, T) = \sum_{n=0}^{2} A_{e,g,n} cos[2\pi n(T - \tau_{e,g,n})], \tag{10}$$

where the parameters to be estimated are the harmonic amplitudes, $A_{e,g,n}$, and the corresponding time lags, $\tau_{e,g,n}$. For the 0^{th} frequency component $A_{e,g,0} = 1$ and $\tau_{e,g,0} = 0$. The $n = 1$ term has an annual period and the $n = 2$ term has a semi-annual period. The semi-annual term produces two peaks within a year and the annual terms introduces an asymmetry between the peaks. This functional form allows for differences in the summer and winter peaks for natural gas and electricity prices. The no-arbitrage forward curve is defined as the product of the secular and the seasonal factors:

$$\begin{aligned} F_{e,g}(T) &= F_{e,g}(S_{e,g}, L_{e,g}, a_{e,g,}, b_{e,g}, A_{e,g,1}, \tau_{e,g,1}, A_{e,g,2}, \tau_{e,g,2}, T) \\ &= \Psi_{e,g}(S_{e,g}, L_{e,g}, a_{e,g}, b_{e,g}, T)\Omega_{e,g}(A_{e,g,1}, \tau_{e,g,1}, A_{e,g,2}, \tau_{e,g,2}, T). \end{aligned} \tag{11}$$

The parameters of $F_{e,g}(T)$ are obtained using maximum likelihood with an objective function that minimizes the sum of squared errors between the observed market prices for electricity and natural gas and the fitted prices.

2.3.1 Calibrating Electricity Forward Curves

For a fixed trade date t, as the time to maturity $(T - t)$ increases the instantaneous volatility of forward prices decays exponentially at a rate α. Our calibration approach assesses the instantaneous volatility of forward prices at trade dates taken as the 15^{th} day of each trade month (or a the closest date to the 15^{th} when this date is a weekend or holiday). More precisely, we uses a small 22-day sample of historical price data around the 15^{th} day of each trade month to calculate its corresponding annualized historical time series of volatility.[12] We calibrate the term structure of instantaneous volatility as a function of maturity, while adjusting for the dampening effect due to the package size. The calibrated parameters α and σ results from regressing the logarithm of the average volatility on time to maturity and seasonal package size. We only include quotes for trading dates after 1/1/2004. This provides us with a large enough sample for each power hub (more than 4 years of daily quotes) and allows us to capture recent events that characterize the market conditions of each power region. Also, because nearer maturity packages are more frequently traded, we only include forward contracts with maturities smaller or equal than 24 months in our calibrations.

In Table 3, we report the estimation of the rate of decay of the term structure of volatities, α_e, and the level of the spot price volatility, σ_e.[13] As shown in the Table 3, there is considerable heterogeneity across the electricity hubs in the fitted values of the speed of mean reversion, α_e, of the exponential Hull-White process and in the volatility, σ_e. The results indicate that overall the higher volatility of forward prices is higher in the Western time zones than it is in the Eastern times, but it is the highest for the forward prices observed in the ERCOT hub. The speeds of adjustment to the long run drift, α_e, are not as differentiated by regions as are the volatilities, however, again the ERCOT hub exhibits a higher speed of adjustment than any of the other over-the-counter markets. The effects of these differences will become more apparent in our discussion below.

[12]It is important to note that our historical forward prices reflect quotes from different seasonal packages. Because of averaging effects, it is expected that the the larger the seasonal package, the lower the volatility. As a result, when calculating the historical time series of volatility for each trade month, we also include the size of the seasonal package as an explanatory variable.

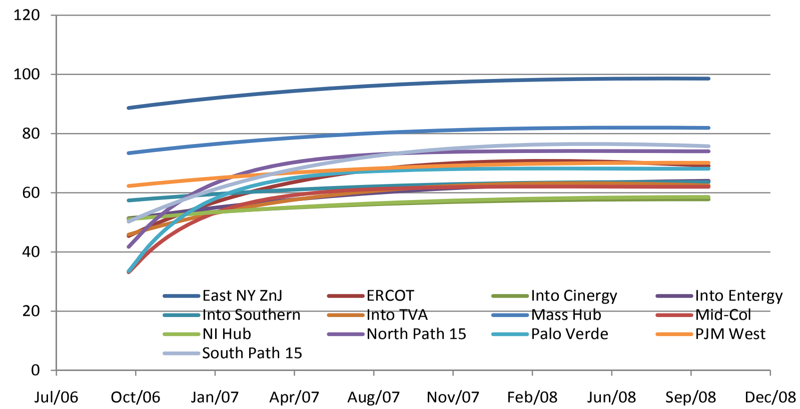
[13]Again, the calibrated parameters α_e and σ_e are estimating by regressing the logarithm of the average volatilities on *month out* measured in years and *package length*.

Table 3: Estimates for the historical values of α_e and σ_e in the Clewlow and Strickland Process for the Electricity Hubs (Average 2004–2010)

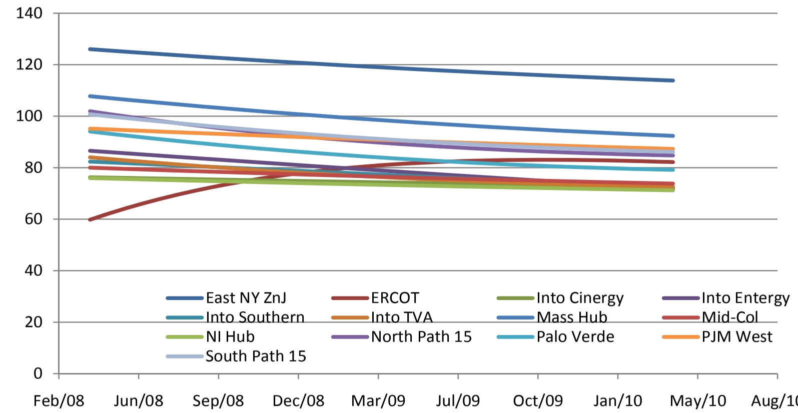
Region	α_e	σ_e
East New York Zone J	0.352	0.313
ERCOT	0.417	0.525
Into Cinergy	0.231	0.384
Into TVA	0.303	0.424
Mass Hub	0.279	0.353
Mid-Columbia	0.175	0.489
Northern Illinois Hub	0.190	0.437
North Path 15	0.236	0.457
Palo Verde	0.206	0.473
PJM Western	0.272	0.347
South Path 15	0.212	0.446

Figure 4 presents a snapshot at four dates: 1) January 1, 2006; 2) April 1, 2008; May 9, 2009; and March 30, 2010; for a cross-section of the fitted secular component of the electricity forward curves for all the electricity hubs in the sample. As is clear from these cross-sections there are some dates, *e.g.* for May 9, 2009 and March 30, 2010, when the forward curves have very similar shapes although the level of prices do differ importantly. Whereas on other dates, *e.g.* for January 1, 2006, the Western hubs appear to move together and for other dates, *e.g.* for April 1, 2008, the ERCOT hub has a significantly different shape. As is clear from these Figures, there is significant heterogeneity in the fitted deseasonalized component of the forward curve both across hubs and between the Eastern, ERCOT, and Western networks, although there are more similarities within each power network. It is also interesting to note that these markets are often decoupled with some hubs exhibiting backwardated (downward sloping) deseasonalized forward curves while at the same time the deseasonalized forward curves for other hubs are in contango (upward sloping). The important differences in the time series dynamics and in the overall level of prices across the various maturities is also quite significant. Overall, Figure 4 strongly suggests that the cross-sectional differences in the risk of electricity exposure should be important in mortgage pricing across regions.

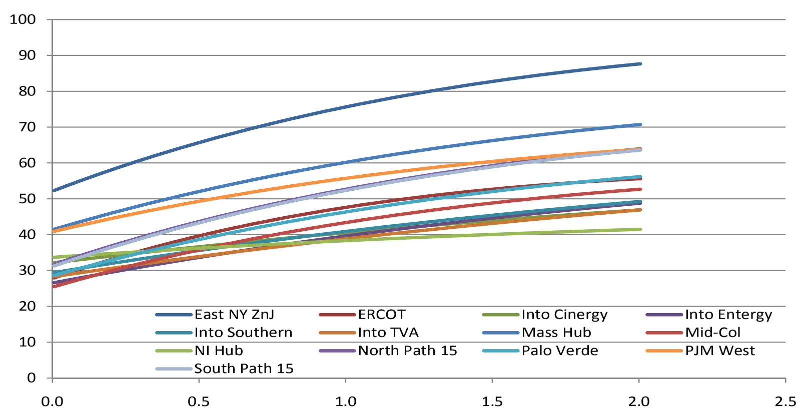
In Figure 5, we present the full forward surface for the ERCOT Hub. As shown in Figure 5 there is a very strong seasonal component to the electricity forward curves. The amplitudes of these seasonals across time reflect weather shocks as well as shocks to supply and demand over the winter and summer months. The most significant weather-related shocks are in the summer months in the ERCOT Hub when the demand for air conditioning is at its highest. Although not shown, the surface plots for the other hubs exhibit similar dynamics, however, there is also considerable variability in the amplitude of the seasonal effects in the cross-section of hubs for specific dates and contract maturities.



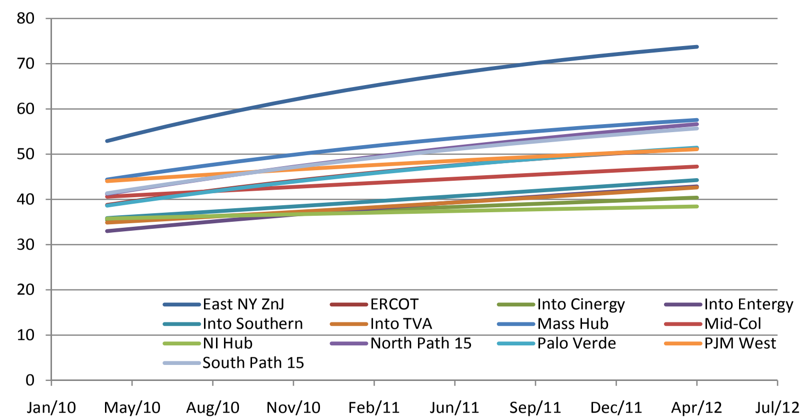
(a) January 1 2006



(b) April 1, 2008



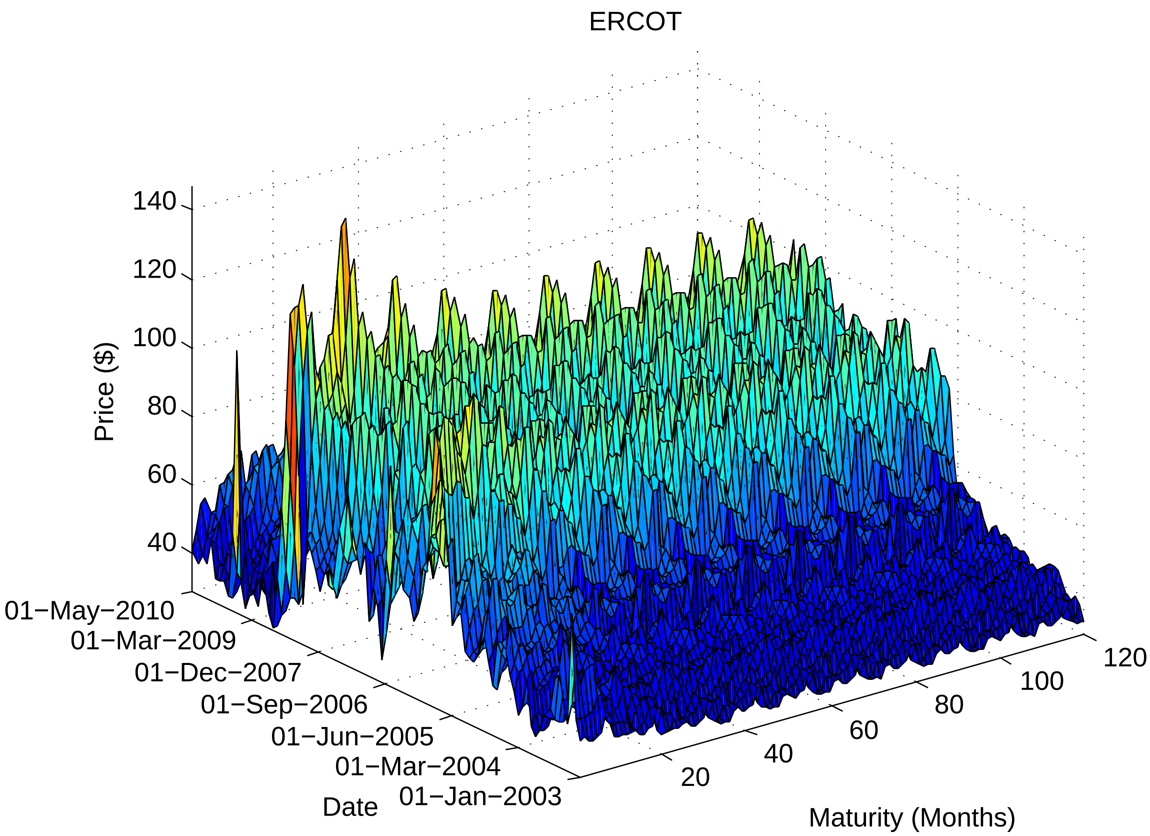
(c) May 9, 2009



(d) March 30, 2010

Figure 4: **Cross-Section of Fitted Deseasonalized Electricity Forward Curves across the Electricity Hubs on Specific dates.** This figure presents a cross section of the secular, or deseasonalized, component of the forward curves for each hub on a single date.

Figure 5: **Estimated ERCOT Electricity Forward Contract Curves**

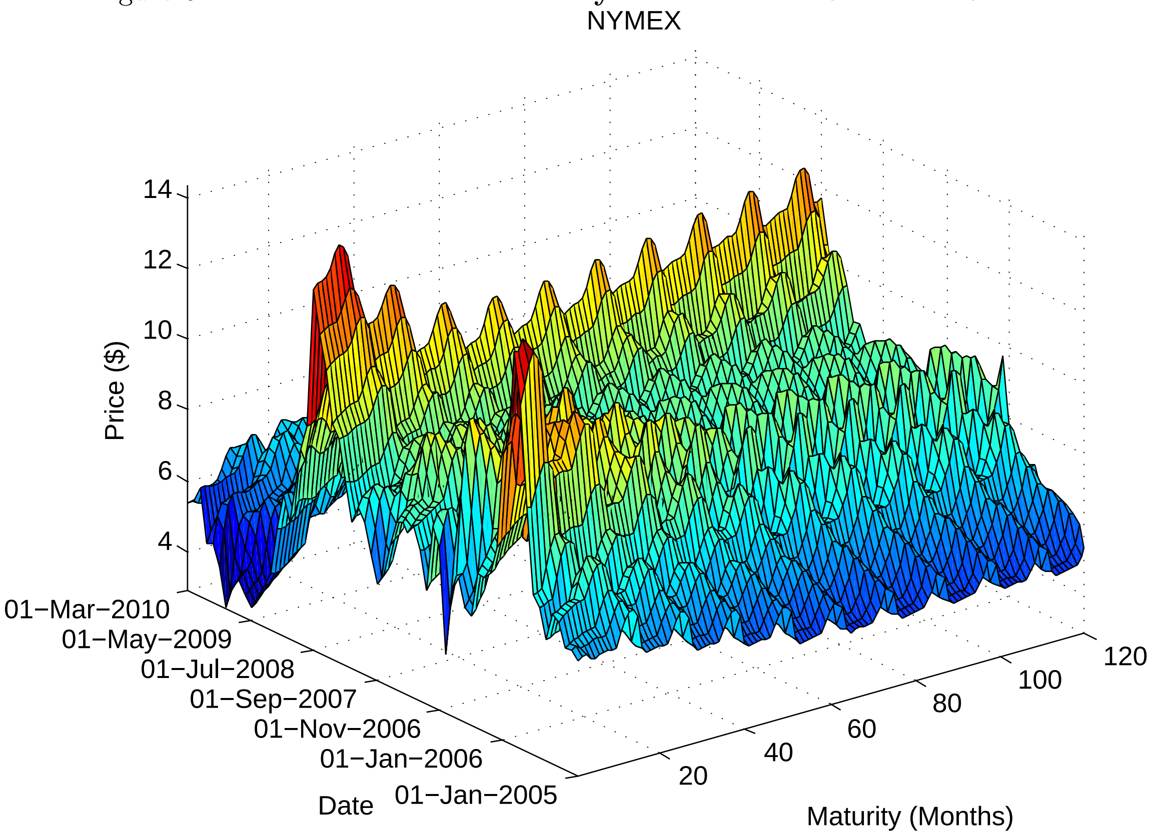


2.3.2 Calibrating the Natural Gas Futures Curves

As previously discussed, there is only one major pricing hub for natural gas, the Henry Hub. As for the electricity hubs, we estimate the parameters for the natural gas spot price dynamics, Equation (6), using data from Henry Hub NYMEX futures and options on NYMEX futures contracts.

In Figure 6, we graph our estimates for the NYMEX Henry Hub futures contract curves over time from 2006 through 2010. As shown there is significant times series variation in the shape and level of the natural gas futures price curve as a function of the maturity of the contracts and again there is a very strong seasonal. As shown, the curves are backwardated in some periods and in contango in others.

Figure 6: **Estimated NYMEX Henry Hub Futures Contract Curves**



3 Two-Part Valuation Strategy

Overall, the fitted factor dynamics for interest rates, energy forward prices, and rents suggest that the energy prices could induce important volatility into cash flows and, therefore into building prices over time. Since commercial mortgage are long contracts, these results indicate that the volatility of energy costs to the building owner could swamp other costs such as janitorial services and the cost of building management staff.

3.1 Part I: Solving for Building-Specific Rental Drift

In order to obtain reliable mortgage values, it is important first to ensure that the valuation model we are using is consistent with the current price of the underlying building. As previously discussed, therefore, in *Stage I* of the valuation strategy on a given date, we fit the interest rate process, the electricity forward process, and the natural gas futures process, then solve for the implied building-specific, risk-adjusted drift for market rents, μ_i, assuming a volatility of 21.478% (see Stanton and Wallace, 2011). The implied drift is the value that makes the valuation model exactly match the observed price of the building at the origination date of the mortgage, given the market dynamics of the three other market fundamentals.

Valuation of the building is performed using Monte Carlo simulation with antithetic variates to estimate the price as the (risk-neutral) expectation of future cash flows,[14]

$$P_t = E_t^* \left[\sum_{k=1}^{\infty} CF_{t+k\,\Delta t} e^{-\Delta t \sum_{j=0}^{k-1} \tilde{r}_{t+j\,\Delta t}} \right]. \qquad (12)$$

Estimating the expectation in Equation (12) involves three steps:

1. Simulate 10,000 paths for rent, interest rates, gas prices, and electricity prices using the risk-neutral processes described above.
2. Calculate the monthly building cash flow (NOI) along each path from Equation (3).
3. Discount each path's cash flows back to the present, and average across all paths.

We repeat this process for various different values of $\hat{\mu}$ in Equation (5), searching numerically until we find the value that makes the building price produced by the Monte Carlo valuation equal to the known price of the building at the mortgage origination date.[15]

3.2 Part II: Solving for Mortgage Value

Valuing mortgages using Monte Carlo simulation is very similar to the process described above for calibrating the risk-neutral drift. Specifically, we start by writing the mortgage value as the risk-neutral expected present value of its future cash flows using Equation (12) again. Then we use Monte Carlo simulation to estimate the expectation the same way as above.

1. Simulate 10,000 paths for rent, interest rates, gas prices, and electricity prices using the risk-neutral processes described above.
2. Calculate the monthly cash flows for the mortgage along each path.
3. Discount each path's cash flows back to the present, and average across all paths.

While structurally similar, there are two significant differences between the two valuations, both related to step 2, the calculation of the mortgage cash flows along each path:

[14] For details see, for example, Boyle (1977); Boyle, Broadie, and Glasserman (1997); Glasserman (2003).

[15] In performing this search, it is important to use the same set of random numbers for each valuation.

1. Commercial mortgages include embedded default options, and when borrowers exercise these options, this affects both the amount and the timing of the mortgage cash flows. To model the borrowers' default behavior, we therefore introduce an empirical hazard model, a model for the estimated conditional probability that a mortgage will default given its survival times, into *Stage II.*
2. Because the likelihood of default at any instant depends on the loan-to-value ratio (LTV), we need a way to estimate the building's value not just at the mortgage origination date, but rather at every date along every path.

We now discuss each of these differences in detail.

3.2.1 Empirical Default Hazard Model

Following standard mortgage-valuation practice (see Schwartz and Torous, 1989), the default hazard for the loans is estimated using a time-varying-covariate hazard model with a log-logistic baseline hazard.[16] Our model also includes controls for loan characteristics including the amortization structure, the loan coupon, amortizing maturity of the loan, the principal due date on the loan, the time varying loan-to-value ratios of the building, and a measure of the difference between the coupon on the loan and the time varying 10 year Treasury rate which is the measure for current interest rates.

We estimate the proportional-hazard model using a sample of 8,497 loans on commercial office buildings that were originated between 2002 and 2007. These data were obtained from Trepp LLC loan-level performance data and include all the origination information on the mortgages along with monthly performance records. The estimated hazard rate is the conditional probability that a mortgage will terminate in the next instant, given that it has survived up until then. Hazard models comprise two components: 1) a baseline hazard that determines the termination rates simply as a function of time and 2) shift parameters for the baseline defined by the time-varying evolution of exogenous determinants of prepayment and default. We define default as a 90-day delinquency on the loan, and model its occurrence via the hazard function

$$\pi(t) = \pi_0(t)e^{\beta\nu}, \qquad \text{where} \tag{13}$$

$$\pi_0(t) = \frac{\gamma p(\gamma t)^{p-1}}{1 + (\gamma t)^p}. \tag{14}$$

The first term on the right-hand side of Equation (13) is the log-logistic baseline hazard, which increases from the origination date ($t = 0$) to a maximum at $t = \frac{(p-1)^{1/p}}{\gamma}$. This is shifted by the factor $e^{\beta\nu}$, where β is a vector of parameters and ν a vector of covariates including the end-of-month difference between the current coupon on the mortgage and U.S. Treasury rates and the current loan-to-value ratio of the mortgage.

The results of our hazard models are reported in Table 4. As expected, there is a statistically

[16] For details on hazard models see, for example, Cox and Oakes (1984).

Table 4: Office Loan-level Estimates for the Default Hazard

	Coeff. Est.	Std. Err.
γ	0.0019***	0.00026
p	1.94387***	0.0898
Current Coupon minus Treasury(t)	0.1613**	0.04561
Loan-to-Value Ratio(t)	0.5771**	0.02225
Number of Observations		8,497

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

significant, positive coefficient on the difference between the coupon rate on the mortgage and the observed 10 year Treasury rate and a statistically significant and large positive coefficient on the current loan-to-value ratio of the loan. Thus, our empirical hazard suggests that loans will default when the difference between the coupon on the loan and the current 10 year U.S. Treasury rate is large and, reasonably, when the value of the loan relative to the value of the building is high.

3.2.2 Empirical Building Value Estimator

In order to estimate default rates along each path, we need an estimate of the building value at every time along each path. In principle, we could just perform a new simulation at every time step along every path, but this would be computationally infeasible. Instead, therefore, we construct an empirical model for building value as a function of current NOI, interest rates and other variables. This is similar to Boudoukh, Richardson, Stanton, and Whitelaw (1997), who used nonparametric regression to estimate the value of mortgage-backed securities as a function of interest rates.[17] Assuming a constant expected growth rate for NOI per square foot and a flat term structure of risk-adjusted discount rates the value, $P(t)$, of an office building per square foot would be given by the Gordon growth model (in logs),

$$\ln P(t) = \ln NOI(t) - \ln(i - g), \tag{15}$$

where g is the market growth rate for net operating income, i is the risk-adjusted discount rate, and we assume $i - g > 0$. This is the basis of our empirical valuation estimator, which we adjust via the inclusion of other explanatory variables.

Based on Equation (15), we fit the following estimator for building values:

$$\ln P(t) = \beta_0 + \beta_1 \ln NOI(t) + \beta_2 \ln i(t) + \beta_3 \ln\left(p_{gas}(t) \times q_{gas}\right) + \beta_4 \ln\left(p_{elec}(t) \times q_{elec}\right), \tag{16}$$

where $\ln P(t)$ is the natural log of the price per square foot of the building on the transaction

[17]We also use this model in the drift calibration. We simulate out to year 10, then use the empirical valuation model to estimate the building's "terminal value" in year 10. This is similar to the use of short-cut methods such as valuation multiples in estimating the terminal value when valuing a business (see Berk and DeMarzo, 2007).

month t, $\ln NOI(t)$ is the natural log of the annual net operating income per square foot on the transaction month t, $\ln i(t)$ is the natural log of the ten-year Treasury rate for the transaction month t, $p_{gas}(t)$ is the average spot price of gas per kBTU for the transaction month t, and q_{gas} is the annual benchmark level of natural gas consumption (kBTU) per square foot for buildings of a corresponding size and location to those reported in Table 2, $p_{elec}(t)$ is average spot price per kWh of electricity for the transaction month t, and q_{elec} is the annual benchmark level of electricity consumption (kWh) per square foot for buildings of a corresponding size and location to those reported in Table 2.

To estimate our building value estimator, we construct a data set that combines two separate transaction data sets: 1) the CoStar Group data; 2) the Trepp LLC data. The Costar data is a comprehensive data set that is maintained by leasing and sales brokers in commercial real estate industry. The data offers comprehensive coverage of transactions across the U.S., although its best geographic coverage is for Western States. We use only the CoStar transactions that are arms-length and confirmed market transactions.[18] The data also include information on the overall building characteristics (building and lot square footage, typical floor area square footage, numbers of floors, etc), how many tenants, the location, and quality characteristics of the building, information on the first and second lien amounts, and the lien periodic payment amounts. For a subset of these data, there is also information on the annual net operating income at sale, the gross rent at sale, and the operating expenses at sales. We then further restricted our sample to the transactions for which we have complete information on transaction characteristics as well as complete information on the annual net operating income at sale, the gross income at sale, and the total annual expenses at sale. This further restriction generated a sample of 1,540 observations from the CoStar transaction data.

Our second data set is obtained from Trepp LLC. Trepp is a data vendor widely regarded as the most accurate source of data on the securitized commercial loan market in the U.S. We restricted the Trepp commercial loan data to those loans that were for transactions and for which we had information on the annual net operating income at sale, the gross rent at sale, and the operating expenses at sales. This restriction leaves us 3,551 transactions. One limitation of the Trepp data is that we only have the underwritten appraised value of the building at the loan origination, rather than the true sales prices. We therefore assume that the appraised value is the market price of the office building. As shown in Table 5 the two samples are not that different. Trepp has slightly more expensive buildings, however, the sample distributions for the revenues and operating expenses levels are comparable for the two data sources. Given this comparability we merge the two data sets together for a total transactions data set of 5,092 observations.

As shown in Table 5 the two data set are quite comparable in revenues and expenses per square foot. The sample of Trepp transactions appear to have sold for a slightly higher price that those

[18] We eliminate all transactions for which there was a "non-arms-length" condition of sale due to such factors as a 1031 Exchange, a foreclosure, a sale between related entities, a title transfer, among other conditions. All of these sale conditions would affect prices due to the trading of tax basis in the case of 1031 exchanges or the auction structure in the case of foreclosure. Instead, we focus only on market transactions between unrelated persons.

Table 5: Sale Transactions Summary Statistics

	N	Mean	Standard Deviation	Minimum	Maximum
CoStar Sample					
Annual Price ($ per Square Foot)	1540	174.20	105.00	6.04	737.25
Annual Revenue ($ per Square Foot)	1540	21.85	9.21	10.00	134.56
Annual Expenses ($ per Square Foot)	1540	7.56	4.17	1	73.11
Trepp Sample					
Annual Price ($ per Square Foot)	3551	205.19	102.81	10.03	872.30
Annual Revenue ($ per Square Foot)	3551	22.11	9.20	10.03	169.55
Annual Expenses ($ per Square Foot)	3551	7.80	3.83	1.05	76.89

Table 6: Sale Transactions Summary Statistics

Variable	N	Mean	Std. Deviation	Minimum	Maximum
Annual Price ($ per Square Foot)	5091	195.81	104.44	6.04	872.30
Annual Revenue ($ per Square Foot)	5091	22.03	9.20	10.00	169.55
Annual Expenses ($ per Square Foot)	5091	7.72	3.94	1.00	76.89
Ten Year Treasury Rate (%)	5091	4.50	0.00	2.90	5.28
Gas Spot Price ($ per kBTU)	5091	0.01	0.00	0.00	0.01
Electricity Spot Price ($ per kWh)	5091	0.07	0.02	0.03	0.16
Building Size (Square Feet)	5091	100350.34	127589.63	15575.00	998770.00
Annual Electricity Consumption (kWh per Square Foot)	5091	1.19	0.35	0.42	3.20
Annual Gas Consumption (kBTU per Square Foot)	5091	6.53	3.89	0.01	75.81

of CoStar. We consider this differences, however, to be minor and we proceed to fit our building value estimator on the joint sample of 5,091 office transactions. The summary statistics for the merged transaction data are presented in Table 6. As shown in Table 6 overall these are fairly large office buildings with an average transaction price of about $195 per square foot. Annual rents were averaged about $22 per square foot and annual operating expenses averaged about $7.7 per square foot. The electricity and gas consumption information for each building was obtained from the CEUS and CBECS benchmark information provided in Table 2 and discussed above.

The results of estimating our building value estimator are reported in Table 7. As shown, the estimator explains about 69% of the observed variance in building prices in the same. As expected the log of net operating income has a statistically significant and positive effect on log price per square foot and the log of the 10 year Treasury rate has a statistically significant and negative effect on log price per square foot. We include the additional covariates to capture the additional effects of energy costs on building transactions prices per square foot. As shown in Table 7, we find that the log of the costs of both natural gas and electricity consumption per square foot have a negative effect on log price per square foot.

Table 7: Estimation Results for the Office Building Valuation using the Trepp and CoStar Merged Data

Variable	Coefficient Estimate	Standard Error
Intercept	2.760***	0.068
Natural Log of Net Operating Income per square foot	0.898***	0.010
Natural Log of the 10 Year Treasury Rate	-7.100***	1.168
Natural log (Gas Spot Price × Gas Consumption)	-0.071***	0.009
Natural log (Electricity Spot Price × Electricity Consumption)	-0.201***	0.017
R^2	0.690	
Includes city fixed effects		

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

4 Valuation Application

Following Figure 3, the valuation of a specific loan requires data for the market term structure of interest rates and volatility, the market energy and natural gas forward curves and their volatility, and the calibrated market rent process for the building. These data need to be collected for the same date, since the model is pricing the mortgage relative to its origination date.

4.1 Loan Valuation Results

To implement our valuation strategy, we first find all of the loans in the Trepp LLC data set that were originated between January 2005 and December 2007, that were within one of the hubs reported in Table 3, and that were located in one of the cities reported in Table 2. An additional selection criterion for a loan to be included in the sample was that Trepp had to have reported the operating expenses and total revenues for the property, the loan was a fixed rate loan, and the "balance due" date of the loan was 120 months. The sample includes 1,390 mortgages located in 28 cities across the U.S. As previously discussed, we fit the Hull-White model for the term structure of interest rates on the origination date of each loan to obtain the estimated values of α_r and σ_r. Similarly, we use the hub-location of the loan to identify the appropriate electricity forward curve for the origination date of the loan. Finally, we fit the forward curve for natural gas NYMEX futures and options at the Henry Hub for all of the origination dates in the sample.[19]

To implement our model, we need information on each of the key contract elements for every loan that will be modeled. As shown in Table 8, across the sample of 1,390 loans there is considerable variety in the square footage of the buildings, the market prices of the buildings, the sizes of the loans, the coupon rates, and the loan-to-value ratios. The overall sample average loan-to-value ratio is about 70.5% and the range of city averages is between 64.53% and 75.18%. The gross rents per square foot in the sample range between \$15.06 and \$27.11 per square foot and there is no apparent

[19] Conditional on the fitted dynamics of interest rates and the electricity and natural gas forward prices, we also fit the drift of the GBM for the building specific rent processes for each loan in *Stage I* of the valuation strategy.

geographic pattern to these rent-levels. As reported in Table 8, the office loans in the sample are quite similar in their amortization maturities. Overall, the loans in the sample are collateralized by larger buildings and the average building size is about eighty-seven thousand square feet with a standard deviation of one hundred and eighty-two thousand square feet. The average building value is about \$17.8 million, with a standard deviation of about \$48.7 million.

As is clear from Table 8, the expected energy consumption by region is highly variable. The reported average gas use by city and electricity hub reflects both the climate of the city and the characteristics of the sample of loans in that city. In the Northeastern and Mid-Western cities, gas is primarily used for winter heating and, as shown in Table 8, buildings in cities such as Cinncinati, Detroit, all of the cities in the Northern Illinois Hub, and Boston are large users of natural gas. Interestingly, buildings in cities such as Phoenix, Miami, Nashville, and Orlando are also shown to be important users of natural gas. Buildings in the ERCOT hub use significantly less natural gas than any other region of the country, however, as shown in Table 8, these buildings are heavier users of electricity which is the primary fuel used in air conditioning.

We carry out five simulation exercises for each loan:

1. *Benchmark Model*

   We fit a benchmark valuation of the loan using the two-factor valuation model of Titman and Torous (1989) (see also Stanton and Wallace, 2011). The two factors in this model are the short-term riskless interest rate, which is assumed to follow the Hull and White model described above, and net operating income, which is assumed to follow the (risk-neutral) geometric Brownian motion process,

   $$dC_{NOI,t} = \hat{\mu}_{NOI} C_{NOI,t}\, dt + \phi_C C_{NOI,t}\, dW_{C_{NOI,t}}. \tag{17}$$

   Here, $C_{NOI,t}$ is net income as a fraction of market value and ϕ_C is the volatility of the property's return, obtained from Stanton and Wallace (2011). Using the procedure discussed above, we calibrate $\hat{\mu}_{NOI}$ loan-by-loan so that the estimated property value equals the observed property value at origination. Having solved for $\hat{\mu}_{NOI}$, we then calculate the value of each mortgage based upon its contract features and using the interest rate model, the dynamics of net operating income, and the empirical hazard model for default discussed above.

2. *Static Model*

   In this set of simulations, we use the interest rate and rental factors. However, while we retain the full information in the forward curves for both electricity and natural gas, we set the volatility of both to zero. The purpose of this set of simulations is to analyze the effect of the level and seasonality of the energy prices, observable from the forward/futures curve, at the origination date of the loan. Presumably, this information could be incorporated into the mortgage contract terms to account for the expected changes in prices over the horizon of the loan which should affect the likelihood of default. We expect that ignoring the forward/futures curves would affect the expected evolution of the net operating income. Since on average the forward/futures curves are upward sloping, we would expect that incorporating this information would lead to valuation discounts relative to the benchmark model.

3. *Stochastic Model*

   In this set of simulations, we use the full four-factor valuation model, accounting for the shape, seasonal components, and time series of volatility of the term structure for electricity forward contracts and natural gas futures contracts. The purpose of this set of simulations is to analyze the effect of the volatility of energy dynamics on mortgage valuations. Given the relative magnitude of electricity and natural gas volatility and the heterogeneity in the volatility of electricity, we would expect this channel to effect the value of the embedded default option in the loans. We expect that the mortgage valuations derived from the full stochastic model would be more discounted than those derived from the static model, although the relative magnitude of these differences are an empirical question.[20]

4. *Stochastic Model with a 20% Reduction in Energy (Both Gas and Electricity)*

   Recent research has shown that it is not unusual to see 10–20% savings in energy consumption in some buildings with very simple energy recommissioning retrofits because the existing operations of many commercial buildings is very inefficient (see Mills, 2009). For this reason, in this set of simulations, we apply the full stochastic model, but assume that there is an immediate 20% reduction in the use of both electricity and natural gas. Here we expect that a downward shift in the consumption of these factor inputs should increase the net operating income of the building, and thus its value. Higher building values would translate into a lowered likelihood that the default option would be exercised by the borrower, thus decreasing the size of the discount in the mortgage valuation relative to the benchmark model.

5. *Stochastic Model with a 20% Reduction in Electricity*

   In this set of simulations, we again apply the full stochastic model, but this time assume that there is an immediate 20% reduction in the use of electricity only. As shown in Table 8, the level of natural gas and electricity consumption varies considerably with the mix of building sizes and across the cities in which the loan is located. The purpose of this set of simulations is to decompose the relative importance of the electricity and the natural gas channels on the valuation of commercial mortgages. We would expect this channel to be importantly determined by location and building size.

We report the results of these mortgage valuations in Table 9. The first two columns of the table report the geographic location of the loan and its electricity hub. The third column reports the average loan value calculated using the benchmark model, and columns 4–7 report the percentage difference between each of the other four specifications and this benchmark value.

It can immediately be seen that each of the four alternative specifications generates mortgage values that are significantly lower than the benchmark model. In addition, the mortgage values increase substantially when electricity or overall power use is reduced by 20%. As shown in the

[20] In our comparisons, of the static and stochastic versions of the valuation model we fix the level of $\hat{\mu}$ to that solved for in the full stochastic version of the valuation framework.

Table 8: Summary Statistics for the Characteristics of the 1,390 Mortgages Used in the Simulations

Electricity Hub	City	Number of Loans	Electricity Use (kWh/sf)	Gas Use (kBTU/sf)	Gross Rents ($/sf)	Other Expenses (sf)	Maturity (Months)	Loan/Value Ratio (%)	Building Price ($M)	Loan Coupon (%)	Building Size (000 Sqft)
Into Cinergy	Atlanta	62	15.19	18.78	19.81	4.96	358.45	73.01	10.64	5.94	62.83
	Charlotte	32	15.13	18.19	17.04	4.10	355.88	72.27	7.67	5.90	53.28
	Cinncinati	36	14.29	40.33	19.35	6.00	366.25	74.03	13.24	5.97	105.00
	Detroit	63	14.08	40.40	20.42	6.38	357.30	73.65	12.48	5.93	85.78
E. New York Zone J	Hartford	40	10.05	36.78	22.92	6.14	362.98	72.67	17.31	5.90	86.26
	N. New Jersey	61	19.02	15.90	23.93	6.10	365.30	69.73	19.10	5.82	82.46
	New York	91	15.74	17.64	27.60	8.04	350.64	71.87	41.39	5.94	144.00
ERCOT	Austin	5	20.18	9.24	25.89	7.91	372.00	75.18	10.94	5.71	47.79
	Dallas	26	22.04	6.09	19.47	6.55	358.62	74.09	29.50	5.79	207.60
	Houston	55	21.79	6.71	17.65	5.82	366.58	73.70	13.27	5.95	108.52
	San Antonio	69	13.45	8.00	18.34	5.77	367.12	71.93	9.21	6.01	73.84
Mass Hub	Boston	29	9.64	35.53	22.93	7.22	361.62	67.29	20.24	5.80	85.37
Mid Columbia	Portland	31	13.70	9.69	19.59	5.19	360.87	66.83	12.51	5.91	78.16
	Seattle	71	13.95	38.84	22.55	6.13	363.34	68.13	14.59	5.89	66.74
N. Illinois Hub	Chicago	49	14.02	40.94	22.20	6.85	351.37	72.47	50.46	5.93	209.16
	Indianapolis	28	15.19	37.28	17.22	5.46	367.64	73.31	14.17	5.92	140.96
	Milwaukee	40	14.43	39.72	17.73	5.15	360.75	69.38	11.29	5.90	67.73
	Minneapolis	42	10.10	36.15	20.12	7.70	361.41	73.20	12.10	6.00	94.11
North Path 15	Sacramento	48	19.13	12.11	24.18	5.68	360.85	65.18	10.14	5.88	46.23
	San Francisco	46	11.67	13.88	27.11	8.21	365.54	64.53	21.49	5.95	78.24
Palo Verde Hub	Phoenix	75	9.38	37.31	21.82	5.67	367.43	70.30	12.48	5.84	60.47
PJM Western	Cleveland	9	17.01	31.88	15.06	5.04	353.33	76.53	17.57	5.81	165.40
South Path 15	Los Angeles	136	15.43	17.51	26.25	7.00	358.60	66.41	17.22	5.80	65.50
	Riverside	33	12.19	14.19	23.15	7.06	364.24	67.65	9.89	5.82	48.01
	San Diego	43	11.48	13.40	26.52	7.50	361.91	64.99	16.31	5.88	59.83
Into TVA	Miami	33	14.17	40.42	22.66	6.86	355.21	68.41	13.67	5.98	69.16
	Nashville	34	9.94	36.41	17.49	5.17	372.88	74.99	9.95	5.87	72.62
	Orlando	77	8.88	36.48	21.16	6.55	364.36	72.82	11.07	5.88	59.72
	St. Louis	26	16.77	15.03	20.30	6.33	369.23	74.58	20.90	5.97	150.13
Average (N=1,390)			14.1	24.9	22.1	6.4	361.6	70.5	17.22	5.9	87.6
Standard Deviation			4.4	14.3	8.1	3.0	35.8	8.7	52.0	0.4	182.0

Table 9: Simulation Results for the 1,390 Mortgages

			Discount to Benchmark Model			
Electricity Hub	City	Benchmark Mortgage Valuation ($ M)	Static Energy Forwards (No Energy Vol.) (%)	Stochastic Energy Forwards (Hub-Defined Vol.) (%)	Stochastic Energy Forwards (20% Decrease in Energy Use) (%)	Stochastic Energy Forwards (20% Decrease in Electricity Use) (%)
E. New York Zone J	Hartford	11.114	5.471	5.555	5.212	5.223
	N. New Jersey	11.010	5.260	5.472	4.938	4.897
	New York	23.034	5.915	6.041	5.601	5.577
ERCOT	Austin	7.597	5.924	6.223	5.791	5.760
	Dallas	19.233	7.707	8.149	7.543	7.483
	Houston	8.858	8.151	8.782	8.049	7.967
	San Antonio	6.230	6.272	6.460	6.094	6.063
Into Cinergy	Atlanta	7.121	4.314	4.446	4.140	4.154
	Charlotte	5.192	4.310	4.498	4.151	4.163
	Cinncinati	8.975	6.448	6.592	6.181	6.237
	Detroit	8.207	5.894	5.983	5.638	5.695
Into TVA	Miami	8.974	4.275	4.375	4.096	4.147
	Nashville	6.851	5.756	5.844	5.514	5.581
	Orlando	7.385	5.052	5.075	4.838	4.899
	St. Louis	14.489	6.677	6.795	6.436	6.415
Mass Hub	Boston	11.232	4.480	4.517	4.262	4.314
Mid Columbia	Portland	7.782	3.445	3.630	3.370	3.357
	Seattle	9.280	3.481	3.663	3.388	3.430
North Path 15	Sacramento	6.327	2.976	3.319	2.974	2.927
	San Francisco	12.972	3.560	3.617	3.434	3.439
N. Illinois Hub	Chicago	29.440	5.351	5.467	5.138	5.188
	Indianapolis	9.397	6.999	7.145	6.683	6.717
	Milwaukee	7.073	4.343	4.548	4.177	4.221
	Minneapolis	7.866	7.437	7.347	7.063	7.117
Palo Verde Hub	Phoenix	7.910	3.657	3.763	3.528	3.569
PJM Western	Cleveland	12.406	8.628	8.847	8.252	8.268
South Path 15	Los Angeles	10.917	3.297	3.482	3.216	3.185
	Riverside	6.219	4.232	4.327	4.088	4.086
	San Diego	10.063	3.102	3.164	2.987	2.969
Average (N=1,390)		10.692	4.968	5.125	4.783	4.791
Standard Deviation		27.297	3.119	3.122	2.988	2.980

last two columns of Table 9, the value of the mortgages on these buildings is now, on average, about 0.34% higher than the same loan on the same building without the retrofit, The size of the difference varies importantly across buildings, mortgage contract structures, and regions. Overall, the reductions in energy consumption appears to benefit the higher loan-to-value ratio mortgages and larger buildings. This result suggests that energy-related retrofits should lower the mortgage rate.

Another pattern that emerges in Table 9 is the importance of regional differences in the level of the discounts in the mortgage valuations relative to the benchmark model. As shown, the highest mean discount levels (highest mispricing relative to the benchmark) are found for loans in the ERCOT hub and for loans in Cleveland and Cinncinati Ohio. The smallest discounts are found in the cities on the West Coast such as San Francisco, Seattle, and Portland. These results suggest that accounting for the heterogeneity in energy efficiency levels across cities and hubs through differential loan contract terms, either loan-to-value ratios or coupons, should be another feature of actuarially fairly priced commercial mortgage loans.

In Table 10, we report the results from regressing the loan valuation discounts from the Benchmark valuations on the cost and net operating income of the building at origination, the size of the building, the loan terms, and the physical location of the building. In the first two columns of Table 10 we present the results from the regressing the loan discounts from the full stochastic model to the benchmark valuation. As shown in the table, the average electricity cost at origination and other non-energy expenses per square foot have a statistically significant and positive effect on the loan discounts. Thus, as expected higher initial costs lead to greater discounts in the loan valuation when the energy effects are accounted for. Again, as expected higher initial net operating income at origination has a statistically significant and negative effect on the discount. The result on building size suggests that larger buildings are less efficient in this sample of loans, since larger buildings have a statistically significantly larger discount.

As shown in Table 10 for the full stochastic model discounts, only two of the initial loan terms have statistically significant coefficient estimates, the origination loan balance and loan-to-value ratio. Higher loan balances are shown to have a negative effect on the size of the discount for the full stochastic model to the benchmark model. As expected, since the loan-to-value ratio is an important trigger of the default options, loans with higher initial loan-to-value ratios have a statistically significant and positive effect on the discount, implying that higher loan-to-value loans appeared to be more mispriced relative to the benchmark model.

All of the regressions reported in Table 10 use Boston (Mass Hub) as the omitted city, so the coefficient estimates for the included indicator variables for cities are the relative to Boston. As shown, loans originated in the cities of Houston and San Antonio in the ERCOT hub, Cleveland in the PJM Hub, and Hartford in the East New York Zone J Hub have statistically significant, at better than the .01 level, higher discounts to the benchmark model valuation using the stochastic model accounting for the level and volatility of energy dynamics in the mortgage valuation. The Dallas, New York city, Indianapolis, and Cinncinati loans also have higher discounts but these

Table 10: Regressions the Percentage of Discount from the Benchmark Model and the Stochastic Model With and Without Efficiency Improvements on Loan Contract Terms and Geography

Origination Characteristics		Percentage of Discount from Benchmark Model					
		Stochastic Energy Forwards		Stochastic Energy Forwards 20% Energy Reduction		Stochastic Energy Forwards 20% Electricity Reduction	
		Coeff. Est.	Std. Err.	Coeff. Est.	Std. Err.	Coeff. Est.	Std. Err.
	Intercept	-8.962***	0.877	-8.512***	0.851	-8.748***	0.834
Cost and NOI Terms	Gas Cost (psf)	0.142	0.631	0.018	0.612	0.148	0.600
	Electricity Cost (psf)	1.095***	0.169	0.850***	0.164	0.824***	0.161
	Other Expenses (psf)	0.616***	0.014	0.619***	0.013	0.616***	0.013
	NOI (psf)	-0.317***	0.007	-0.302***	0.007	-0.303***	0.007
	Property Value	-0.000000003	0.000	-0.000000003	0.000	-0.000000002	0.000
Building Characteristics	Building Square Footage	0.000007***	0.000	0.000006***	0.000	0.000006***	0.000
Loan Terms	Loan Coupon	0.040	0.112	0.029	0.109	0.046	0.107
	Loan-to-Value Ratio	0.188***	0.005	0.179***	0.004	0.182***	0.004
	Original Loan Balance	-0.00000003***	0.000	-0.00000004***	0.000	-0.00000004***	0.000
	Loan Maturity	-0.001	0.001	-0.001	0.001	-0.001	0.001
Origination Year	Year 2006	0.190	0.118	0.131	0.114	0.113	0.112
	Year 2007	-0.043	0.122	-0.081	0.119	-0.099	0.116
E. New York Zone J	Hartford	0.814***	0.310	0.804***	0.301	0.749**	0.294
	N. New Jersey	0.328	0.345	0.325	0.335	0.279	0.328
	New York	0.514*	0.305	0.493*	0.296	0.448	0.290
ERCOT	Austin	0.294	0.630	0.265	0.612	0.212	0.599
	Dallas	0.675***	0.371	0.592	0.360	0.521	0.353
	Houston	1.546***	0.330	1.361***	0.321	1.260***	0.314
	San Antonio	0.798***	0.314	0.786***	0.305	0.721**	0.299
Into Cinergy	Atlanta	-0.157	0.299	-0.119	0.290	-0.158	0.284
	Charlotte	-0.010	0.341	0.022	0.331	-0.013	0.324
	Cinncinati	0.535*	0.318	0.506	0.309	0.492	0.303
	Detroit	0.009	0.288	0.039	0.279	0.023	0.273
Into TVA	Miami	-0.166	0.322	-0.157	0.312	-0.163	0.306
	Nashville	0.318	0.323	0.325	0.313	0.308	0.307
	Orlando	0.174	0.279	0.178	0.271	0.162	0.266
	St. Louis	0.470	0.356	0.503	0.346	0.444	0.339
Mid Columbia	Portland	0.015	0.356	0.010	0.346	-0.028	0.339
	Seattle	-0.088	0.279	-0.104	0.270	-0.122	0.265
N. Illinois Hub	Chicago	0.121	0.300	0.109	0.291	0.088	0.286
	Indianapolis	0.660*	0.340	0.637*	0.330	0.606*	0.323
	Milwaukee	-0.235	0.311	-0.250	0.302	-0.268	0.295
	Minneapolis	0.583*	0.310	0.603**	0.301	0.585	0.294
North Path 15	Sacramento	0.208	0.331	0.207	0.321	0.154	0.315
	San Francisco	0.187	0.322	0.171	0.313	0.161	0.306
Palo Verde Hub	Phoenix	0.092	0.278	0.092	0.270	0.064	0.264
PJM Western	Cleveland	1.355***	0.485	1.258***	0.471	1.209	0.461
South Path 15	Los Angeles	0.157	0.278	0.146	0.270	0.099	0.264
	Riverside	0.059	0.341	0.062	0.331	0.035	0.325
	San Diego	0.212	0.329	0.189	0.319	0.154	0.313
R^2		0.84		0.84		0.84	
N = 1,390							

t statistics in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

coefficient estimates are statistically significant at only the .10 level or better. As reported in Table 8, the buildings in the ERCOT hub cities are the heaviest users of electricity per square foot. Again from Table 8, buildings located in Cleveland, Indianapolis, Cinncinati, and Hartford are significant users of natural gas relative to the rest of the buildings in the sample. These results suggest that the building energy consumption characteristics and the environmental conditions of these cities are not being well accounted for using valuation models that ignore the level and dynamics of electricity and natural gas prices.

In Columns 5 through 8 of Table 10, we report the effect on the discounts to the benchmark model of a permanent 20% reduction (at the origination date) in the buildings consumption of natural gas and electricity and a permanent 20% reduction of electricity only. As shown, the same factors as before are shown to have a statistically significant effect on the discount, but the size of the discount due to the loan-to-value ratio and the electricity costs is reduced. The discounts in the loan valuations that are associated with building size inefficiencies are unchanged by shocks to use levels for either both natural gas and electricity or for electricity only. As shown in Table 10, the discounts are reduced for Hartford, New York, Dallas, Houston, San Antonio, Cinncinati, Indianapolis, and Cleveland, but these benefits are not statistically different from zero for New York and Cinncinati where the buildings are heavy consumers of natural gas. The efficiency gains for Hartford, Houston, San Antonio, and Cleveland are all statistically significant for reductions in both gas and electricity and, interestingly, are of larger magnitude for the reductions in electricity only. Surprising, the effects on the discounts to the benchmark for loans originated in Dallas are of the right sign but they do not remain statistically significant.

Overall these results suggest that including the effects of energy dynamics is an important component of the mortgage valuation problem. Given the significant heterogeneity in the building-level consumption of natural gas and electricity across the country and the important heterogeneity in the forward/futures price dynamics for these commodities by region, these results strongly suggest that energy should affect the cost of capital of the first lien positions. For this reason, developing first lien underwriting models to price this risk is of significant importance.

5 Conclusion

In this paper, we develop a commercial-mortgage valuation, or underwriting, strategy that accounts for the energy risk of individual office buildings in terms of the energy efficiency of the buildings and the characteristics of their locations. Our method extends standard underwriting practices, which account for the expected dynamics of interest rates and office building prices over time, by including the expected dynamics of the electricity and gas prices as well as quantity dynamics appropriate to the location of the building. Our proposed method allows lenders to explicitly take into account the effect of energy use and various alternative efficiency measures when underwriting commercial mortgages. We find that, relative to the traditional modeling strategy, our proposed strategy leads to an 5% reduction in the mispricing of the default risk of commercial mortgages.

The valuation framework can also be applied to price the benefits of energy retrofits, in terms of their effect on the relative risk of commercial mortgage.

Overall, the valuation method has been shown to be tractable for actual market applications to price real mortgage products even without application of the engineering reports that are already part of the commercial mortgage underwriting process. We have also shown that benchmarks for the energy consumption of office buildings determined by the square footage of the building and its location are adequate to differentiate the relative energy risk of commercial mortgages. Further development of efficiency metrics in conjunction with further refinements in our mortgage valuation framework will assure that the energy risk of commercial office buildings can be assessed as a matter of course in the mortgage underwriting process.

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