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Employee Stock Option Exercise and Firm Cost

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ABSTRACT

We develop an empirical model of employee stock option exercise that is suitable for valuation and allows for behavioral channels. We estimate exercise rates as functions of option, stock, and employee characteristics using all employee exercises at 88 public firms, 27 of them in the S&P 500. Increasing vesting frequency from annual to monthly reduces option value by 11% to 16%. Men exercise faster, reducing value by 2% to 4%, while top employees exercise slower, increasing value by 2% to 7%. Finally, we develop an analytic valuation approximation that is more accurate than methods used in practice.

EMPLOYEE STOCK OPTIONS (ESOs) ARE A major component of corporate compensation and a material cost to firms. Murphy (2013) reports that options represented 21% of CEO pay in 2011, and Meridian Compensation Partners (2018) report that 42% of their national sample use executive stock options. Despite the overall importance of ESOs in executive compensation, their valuation remains a challenge in practice as shown by the recent controversy over the value of options granted to the CEO of IBM, which Institutional Shareholder Services valued at more than twice the company's valuation.[1] The valuation of long-maturity American options such as ESOs is sensitive to assumptions about how they will be exercised. Because employees face hedging constraints,

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[1] See Melin, Anders, 2017, IBM says CEO pay is $33 million. Others say it is far higher, https://www.bloomberg.com/news/articles/2017-04-24/ibm-says-ceo-pay-is-33-million-others-say-it-is-far-higher.

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standard option exercise theory does not apply.[2] In addition, commonly used modifications to Black-Scholes (1973) are inadequate because they ignore key empirical features of employee exercise patterns.

In this paper, we develop an empirical model of employee option exercise suitable for valuation and apply it to a sample of all employee exercises at 88 publicly traded firms from 1981 to 2009. We develop a Generalized Method of Moments (GMM)-based methodology that is robust to both heteroskedasticity and correlation across option exercises to estimate the rate of voluntary exercise as a function of the stock price path and of firm, contract, and option holder characteristics. We show that these characteristics significantly affect exercise behavior, in a manner consistent with both portfolio theory and results from the behavioral literature, such as Barber and Odean (2001). Our data are provided by corporate participants in a sponsored research project funded by the Society of Actuaries in response to regulatory calls for improved ESO valuation methods.

Using the estimated exercise function, we value a range of options and find that vesting structure has a large effect on the option cost to firms: increasing vesting-date frequency from annual to monthly reduces option value by as much as 16% for typical option holders. In addition, men exercise options significantly faster than women, resulting in a 2% to 4% lower option value. Higher ranked option holders exercise more slowly than lower ranked option holders, implying that the options of top employees are typically worth 2% to 7% more than those of lower ranked employees. Top employees are less responsive to the passage of vesting dates and the stock price reaching a new high. Consistent with theory, employees with greater wealth tend to exercise more slowly, and employees with greater option risk tend to exercise faster. Consistent with both standard option theory and utility maximization, the rate of voluntary option exercise is positively related to the level of the stock price and the imminence of a dividend. However, the exercise rate is also higher when the stock price is in the 90th percentile of its distribution over the past year, which may reflect behavioral effects.

We compare option values calculated using our model and Black-Scholes-based methods commonly used in practice. Even when their inputs are estimated perfectly, we find that the Black-Scholes-based approximations have significant biases, which vary systematically with firm characteristics.[3] Given the practical appeal of a Black-Scholes-based approximation, however, we develop an alternative Black-Scholes approximation to our model that matches the correct option values more closely than existing methods. It is easy to

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[2] For example, employees systematically exercise options on nondividend-paying stocks well before expiration, and this substantially reduces their cost to firms.

[3] Moreover, for dividend-paying firms, it is possible for the true (American) option value to be so large that the (European) Black-Scholes formula *cannot* give the correct result. To illustrate the problem, a 10-year American call option with strike price \$100 on a stock with $S = 100$, $r = 0.05$, $\sigma = 0.3$, and a dividend rate $d = 0.06$ has value \$24.59. With the same parameters and varying the expiration date, the highest possible European option value is \$18.72, which occurs at an option maturity of 7.44 years.

implement analytically without the need for additional data, can serve as the basis for a new accounting valuation method, and can also be used in corporate finance research on executive and employee compensation.

The paper proceeds as follows. Section I briefly summarizes prior literature in this area. Section II describes our valuation and estimation methodology and develops an empirical model of employee exercise that is both flexible enough to capture observed exercise behavior and suitable as a basis for option valuation. Section III describes the sample of proprietary data on employee options. Section IV presents estimation results for the empirical exercise function. Section V analyzes the impact of the estimated exercise factor effects on employee option cost and the approximation errors of current valuation methods, and develops a new analytic approximation method. Section VI concludes.

I. Previous Literature

The principles of employee option valuation and the need to study exercise behavior are well understood. One approach taken in the literature is to model the exercise decision theoretically. The employee chooses an option exercise policy as part of a greater utility maximization problem that includes other decisions such as portfolio, consumption, and effort choice, and this typically leads to some early exercise for the purpose of diversification. Papers that develop utility-maximizing models and calculate the implied cost of options to shareholders include Huddart (1994), Detemple and Sundaresan (1999), Ingersoll (2006), Leung and Sircar (2009), Grasselli and Henderson (2009), and Carpenter, Stanton, and Wallace (2010).

Combining theory and data, papers such as Carpenter (1998) and Bettis, Bizjak, and Lemmon (2005) calibrate utility-maximizing models to mean exercise times and stock prices in the data, and then infer option value. However, these papers provide no formal estimation and the approach relies on the validity of the utility-maximizing models used. Huddart and Lang (1996), Heath, Huddart, and Lang (1999), and Klein and Maug (2011) provide more flexible empirical descriptions of option exercise patterns, but do not go as far as option valuation. Armstrong, Jagolinzer, and Larcker (2007) perform a valuation based on a hazard model of the exercise of option grants, but this specification is inappropriate for valuation because employees exercise random fractions of outstanding option grants.

A number of analytic methods for approximating employee option value have also been proposed. FAS 123R permits using the Black-Scholes formula with the expiration date replaced by the option's expected life, and SAB 110 permits using Black-Scholes with expiration replaced by the average of the contractual vesting date and expiration date. Jennergren and Näslund (1993), Carr and Linetsky (2000), and Cvitanić, Wiener, and Zapatero (2008) derive analytic formulas for option value assuming exogenously specified exercise boundaries and stopping rates. Hull and White (2004) propose a model in which exercise occurs when the stock price reaches an exogenously specified multiple of the stock price and forfeiture occurs at an exogenous rate. However, until the accuracy of these methods can be determined, their usefulness cannot be assessed.

In contrast to the existing academic literature, our proprietary data set contains all employee exercises at a large number of firms.[4] Since estimation of exercise behavior requires a large sample of stock price paths, and thus a large number of firms, we are able to investigate differential option exercise patterns and option costs across top-ranked and lower ranked employees.

II. Valuation and Estimation Methodology

Like structural models of optimal corporate default and mortgage prepayment, structural models of optimal employee option exercise provide insights about which variables drive exercise decisions and why.[5] However, for the purpose of estimating option values from the data, more flexible, reduced-form models are necessary to describe exercise behavior in large pools of heterogeneous option holders in settings that are more complex than is tractable in theory. It seems natural to use hazard rates (see, e.g., Kalbfleisch and Prentice (1980), Cox and Oakes (1984)) to model employee option exercise, since they are often used in the finance literature to model somewhat similar events, such as mortgage prepayment, as in Schwartz and Torous (1989), and corporate bond default, as in Duffie and Singleton (1999). However, hazard models do not adequately accommodate the fact that employees typically exercise grants of options partially and repeatedly, as predicted by theory.[6] For example, to estimate hazard models of employee option exercise, Armstrong, Jagolinzer, and Larcker (2007) and Klein and Maug (2011) identify exercise events as those in which the fraction exercised exceeds a prespecified threshold. This can lead to biases when the hazard model of exercise is passed into an option valuation model. Similarly, Ordinary Least Squares (OLS) models of the fraction of available options exercised, such as in Heath, Huddart, and Lang (1999), are unsuitable as an input to valuation because such models do not restrict the fraction to lie between 0 and 1.

To avoid these problems with prior approaches, we model the expected fraction of the remaining vested in-the-money options exercised by option holder i from grant j on day t as a logistic function of a vector of covariates X_t that theory and previous empirical studies suggest should influence exercise decisions:

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[4] For example, the samples of Heath, Huddart, and Lang (1999) and Armstrong, Jagolinzer, and Larcker (2007) contain only 7 and 10 firms, respectively, while the sample of Klein and Maug (2011) contains only top executives.

[5] See, for example, Merton (1973), Van Moerbeke (1976), Roll (1977), Geske (1979), Whaley (1981), and Kim (1990) for standard American option models of the value-maximizing exercise policy. See Huddart (1994), Marcus and Kulatilaka (1994), Carpenter (1998), Detemple and Sundaresan (1999), Ingersoll (2006), Leung and Sircar (2009), Grasselli and Henderson (2009), and Carpenter, Stanton, and Wallace (2010) for utility-maximizing models of option exercise that account for trading constraints on option holders.

[6] See Grasselli and Henderson (2009).

$$G(X_{ijt}\beta) = \frac{\exp(X_{ijt}\beta)}{1+\exp(X_{ijt}\beta)} \,. \tag{1}$$

The specific covariates included in X_{ijt}, described in detail in Section III.C, include the ratio of the stock price to the strike price and other stock, employee, and option grant characteristics. For unvested options, and for vested options that are out of the money, we define $G \equiv 0$.

The exercise rate function $G(X_{ijt}\beta)$ is the probability that an individual option in the grant will be exercised at date t, conditional on the time, stock price, and other state variables described by X_{ijt}, and conditional on having survived to time t. Assuming that residual uncertainty about the rate of exercise around $G(X_{ijt}\beta)$ is not priced,[7] we can also regard $G(X_{ijt}\beta)$ as the risk-neutral probability of voluntary exercise. We similarly define the employee termination rate, λ, which describes the probability that the option will be involuntarily stopped through termination, forcing exercise of vested in-the-money options, cancellation of vested out-of-the-money options, or forfeiture of unvested options.

Given a voluntary exercise rate function $G(X_{ijt}\beta)$ and employee termination rate λ, the value of the option is given by its expected discounted payoff under the risk-neutral probability measure. In a grant that vests in fractions $\alpha_1, \ldots, \alpha_n$ at times $t_1, \ldots, t_n$, the average option value is[8]

$$\begin{aligned} O_t = \sum_{k=1}^{n} \alpha_k \mathrm{E}_t^* \Bigg\{ & \int_{t\vee t_k}^{T} e^{-r(\tau-t)} (S_\tau - K)^+ (G_\tau + \lambda) e^{-\int_t^\tau (G_s+\lambda)\,ds}\, d\tau \\ & + e^{-r(T-t)} e^{-\int_t^T (G_s+\lambda)\,ds} (S_T - K)^+ \Bigg\}. \end{aligned} \tag{2}$$

This is the cost of the option to the firm, as opposed to the subjective value of the option to the employee, who cannot trade the option.[9] It involves averaging across possible stock price paths and across possible exercise times, where the weight $(G_\tau + \lambda)e^{-\int_t^\tau (G_s+\lambda)\,ds}$ in the integrand above can be viewed as the probability of exercise at time τ, conditional on the stock price path and on having survived until τ. For a given exercise function G and termination rate λ, O_t can be computed using Monte Carlo simulation, as described in more detail in the Appendix.

We estimate the parameter β in the conditional exercise rate $G(X_{ijt}\beta)$ using the fractional-logistic approach of Papke and Wooldridge (1996). Let y_{ijt} be the

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[7] This would be the case, for example, if any residual uncertainty diversifies away in a large enough pool of option holders.

[8] While $G(X_{ijt}\beta)$ defined in equation (1) refers to the daily exercise rate, G in equation (2) refers to the annualized exercise rate.

[9] Studies of subjective option value to the employee include Lambert, Larcker, and Verrecchia (1991), Hall and Murphy (2002), Ingersoll (2006), Bergman and Jenter (2007), Carpenter, Stanton, and Wallace (2010), and Murphy (2013).

fraction of remaining vested in-the-money options held by individual i from grant j that is exercised on day t, and write

$$y_{ijt} = G(X_{ijt}\beta) + \epsilon_{ijt}, \tag{3}$$

where X_{ijt} is a set of covariates in I_t, the information set at date t, and where

$$E(\epsilon_{ijt} \mid I_t) = 0,$$
$$E(\epsilon_{ijt}\,\epsilon_{i'j't'}) = 0 \quad \text{if } i \neq i' \text{ or } t \neq t'.$$

Note that while we are assuming that the residuals ϵ_{ijt} are uncorrelated between individuals and across time periods, we are allowing ϵ_{ijt} to be arbitrarily correlated between different grants held by the same individual at a given point in time, and we are not making any further assumptions about the exact distribution of ϵ_{ijt}, or even about its variance.[10] In particular, rather than assume a beta distribution for y_{ijt} (see Mullahy (1990), Ferrari and Cribari-Neto (2004)), we are allowing for a strictly positive probability that y_{ijt} takes on the extreme values 0 or 1.

We estimate the parameter vector β using quasi-maximum likelihood (see Gouriéroux, Monfort, and Trognon (1984)) with the Bernoulli log-likelihood function,

$$l_{ijt}(\beta) = y_{ijt}\log\left[G(X_{ijt}\beta)\right] + (1 - y_{ijt})\log\left[1 - G(X_{ijt}\beta)\right]. \tag{4}$$

With standard errors clustered by employee-day (see Rogers (1993), Baum, Schaffer, and Stillman (2003), Wooldridge (2003), Petersen (2009)), our method generates consistent estimates of expected exercise rates that are guaranteed to lie between 0 and 1, it allows for correlation between option exercises from different grants held by the same individual, and it allows for arbitrary heteroskedasticity in the exercise rates.

III. Data and Variable Construction

Our estimation is carried out using a proprietary data set that contains detailed option data for all employees in a large sample of firms. The data, which comprise an unbalanced sample from 1981 to 2009, contain complete histories of ESO grants, vesting structures, and option exercise, cancellation, and termination events for all employees who received options at 88 publicly traded corporations.[11] This data set gives us an unprecedented level of detail on option exercise behavior, including data on over 290,000 employees, over 810,000 employee option grants, and more than 560,000 different exercise events. As

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[10] While we do not explore this further, residuals could also be correlated across different individuals at the same firm at the same point in time, since an individual who exercises his or her options might mention this to a colleague, increasing the likelihood that the colleague will also exercise.

[11] The data were obtained as part of a research grant written by the authors and funded by the Society of Actuaries.

described by equation (3), the dependent variable is the fraction of remaining vested in-the-money options held by an employee from a given grant that is exercised on a given trading day. Its conditional mean is modeled as a logistic function of stock, employee, and option grant characteristics, as in equation (1). From our detailed data set, we are able to construct a sample of over 378 million employee-grant-day observations from which we estimate this conditional exercise rate function.

A. *Sample Firms*

Table I presents summary statistics for the sample firms. As the table shows, there is considerable heterogeneity in the sample of firms in terms of their industry (reported at the one-digit Standard Industrial Classification [SIC] code level), firm size (as measured by market capitalization and number of employees), net income, revenue, and revenue growth over the period. The differences in the mean and median values for the reported performance statistics are also indicative of heterogeneity in the sample. As an indication of the representativeness of our sample overall, there are 27 S&P 500 firms. The sample includes relatively more technology firms (SIC 3), and fewer utilities firms (SIC 4), consistent with the cross-sectional pattern of option use by firms reported in Core and Guay (2001). Firm size tends to be larger in the transportation and utility sector (SIC 4) and retail sector (SIC 5), and is smallest in the hospital and educational services sectors (SICs 8 and 9). Market-to-book ratios are also quite different across these industries, with technology firms (SIC 3) exhibiting the highest average market-to-book ratios at 12.77, perhaps reflecting the importance of their relative growth options. The transportation and communications sector (SIC 4) and the finance, insurance, and real estate firms (SIC 6) have the lowest market-to-book ratios. The low values for industries in SIC 4 probably reflect the higher levels of infrastructure capital found in these firms. The low values for firms in SIC 6 possibly reflect the expected low residual value of these firms in default. The industries with the highest realized growth rates are construction and manufacturing firms (SICs 1 and 2), with growth rates of 0.45 over the period, and computer services firms (SIC 7), with growth rates of 1.47 over the period. Clearly, Table I suggests that controls must be introduced for the important heterogeneity in the cross-section of industry and firm types in the data.

B. *Option Grants and Exercises*

Panel A of Table II summarizes the size and structure of the sample of option data both by industry and in aggregate over all the firms in the sample. In total, there are 561,073 option exercises across 810,348 grants to 292,052 employees in the 88 sample firms. As shown in Panel B, on average there are 2.17 grants per employee and the median number of grants is 1.0. There is considerable variability across firms and employees, however, with some employees

Table I
Summary Statistics for Sample Firms (1981 to 2009)

This table provides financial performance summary statistics for the 88 firms in our sample, grouped by one-digit SIC code. Employees is the sample-period mean of the annual number of employees for the firms in that one-digit SIC, reported in Compustat as "*Employees: Thousands*" (DATA29). Market cap. is the sample-period mean of "*Common Shares Outstanding: Millions*" in Compustat (DATA25) multiplied by "*Price minus Fiscal Year Close*" in Compustat (DATA199). Market-to-book is the sample-period mean of market capitalization scaled by common equity as reported in Compustat (DATA60). Revenue is the sample-period mean of "*Sales/Turnover*" in Compustat (DATA12). Net income is the sample-period mean of "*Net Income*" in Compustat (DATA172).

	Construction and Manufacturing		Transportation, Communications, and Utilities	Retail	Finance, Insurance, and Real Estate	Services		All
One-Digit SIC	1 and 2	3	4	5	6	7	8 and 9	
Number of firms	13	25	3	13	13	14	7	88
Number of S&P 500 firms	1	7	2	6	6	5	0	27
Employees (000)								
Mean	3.12	6.05	19.15	24.95	12.50	10.49	3.57	10.32
Median	0.29	1.77	12.54	12.56	4.52	3.18	3.56	3.56
Market cap. ($M)								
Mean	652.68	1,983.20	9,578.27	2,830.54	5,290.98	2,092.21	646.31	2,570.39
Median	404.65	801.65	2,873.65	994.24	1,842.64	936.48	835.39	821.49
Market-to-book								
Mean	3.84	12.77	2.52	4.22	2.34	6.19	7.30	6.81
Median	4.11	3.65	2.59	3.44	1.92	5.00	6.32	3.84
Revenue ($M)								
Mean	483.73	865.27	3,446.75	5,314.29	3,723.68	1,211.94	355.50	1,991.02
Median	112.65	310.67	1,637.35	1,939.50	696.96	532.87	305.50	476.27
Revenue growth (%)								
Mean	0.45	0.23	0.13	0.16	0.13	1.48	0.12	0.42
Median	0.18	0.16	0.08	0.16	0.09	0.10	0.10	0.12
Net income ($M)								
Mean	10.95	49.32	250.82	106.91	314.57	55.99	22.23	97.12
Median	−10.87	6.06	148.60	39.66	52.16	11.13	19.63	30.21

Table II

Summary Statistics for Employee Option Grants, Vesting Structures, and Exercises

This table provides summary statistics within industrial groupings for employee option grants, vesting structures, and exercise patterns.

	Construction and Manufacturing		Transportation, Communication, and Utilities	Retail	Finance, Insurance, and Real Estate	Services		All
One-Digit SIC	1 and 2	3	4	5	6	7	8 and 9	
Panel A: Aggregate Number of Employees, Grants, and Exercise Events								
Number of employees	5,443	75,102	11,955	34,220	110,680	48,385	6,267	292,052
Number of grants	20,584	249,190	18,736	116,955	254,269	137,086	13,576	810,348
Number of exercises	16,947	146,183	5,366	99,142	179,615	106,255	7,595	561,073
Panel B: Number of Grants per Employee								
Mean	3.78	3.33	1.56	3.42	2.97	2.83	2.16	2.17
Median	3.00	2.00	1.00	2.00	1.00	2.00	2.00	1.00
SD	3.68	1.41	1.41	3.19	2.36	2.98	2.00	2.01
Panel C: Dollar Value of Underlying Shares per Grant ($ Thousands)								
Mean	90.27	46.15	106.32	52.64	106.41	59.34	40.05	65.51
Median	25.56	13.65	52.32	11.51	25.85	26.89	11.60	16.65
SD	379.99	263.48	321.81	299.67	715.94	243.27	158.38	425.66
Panel D: Number of Vesting Dates per Grant								
Mean	9.25	6.90	1.30	2.70	2.95	4.36	3.14	4.38
Median	4.00	4.00	1.00	2.00	3.00	4.00	2.00	3.00
SD	14.55	9.30	0.86	1.84	1.36	3.42	1.25	6.52

(Continued)

Table II—*Continued*

	Construction and Manufacturing		Transportation, Communication, and Utilities	Retail	Finance, Insurance, and Real Estate	Services		All
One-Digit SIC	1 and 2	3	4	5	6	7	8 and 9	
Panel E: Percentage of Options That Vest on the First Vesting Date (per Grant)								
Mean	42.77	37.16	90.38	61.69	48.32	38.54	37.63	51.28
Median	25.00	25.00	25.00	33.33	33.33	25.00	50.00	33.33
SD	31.82	30.48	20.67	38.64	29.54	29.75	14.75	35.63
Panel F: Number of Months from Grant to Full Vesting (per Grant)								
Mean	36.39	44.76	44.44	28.13	33.80	40.49	37.74	56.93
Median	48.00	48.00	48.00	36.00	36.00	48.00	24.00	38.00
SD	15.78	20.34	5.48	21.31	19.75	20.87	15.72	25.56
Panel G: Number of Months from Grant to Expiration (per Grant)								
Mean	113.06	72.00	120.00	122.40	120.18	120.00	120.00	120.29
Median	120.00	120.00	120.00	120.00	120.00	120.00	120.00	120.00
SD	19.81	5.48	0.00	12.63	0.38	0.00	0.00	6.42
Panel H: Fraction of Grant's Vested Options That Are Exercised (per Exercise Event)								
Mean	0.72	0.84	0.58	0.68	0.58	0.57	0.93	0.67
Median	1.00	1.00	0.67	0.81	0.62	0.51	1.00	0.95
SD	0.35	0.29	0.22	0.35	0.40	0.39	0.18	0.38

managing dozens of option grants. For the firms that reported employee titles, the largest grant recipients are typically the CEO or senior managers.

As shown in Table II, Panel C, the average grant has $65,510 worth of underlying shares at the grant date, but this varies widely across industries. As can be seen, the highest mean and variance for grant size is found in the finance industry, where the average value is $106,410 worth of underlying shares at the grant date. The median grant has $16,650 worth of underlying shares. The combined effect of the potentially large number of grants per employee and the size of these grants implies that individual employees may hold large inventories of options with different strikes, expiration dates, and vesting structures. Thus, there is likely to be high correlation in exercise decisions across grants held by the same individual. A particular strength of our fractional-logistic estimator is that it does not require assumptions of independence across exercise events.

The vesting structures also vary widely, both across and within firms in our sample, and can be complex. As shown in Panel D, the average grant has 4.38 vesting dates and the median is 3.00, but this varies significantly by one-digit SIC code. In our data, we see vesting structures with as many as 60 vesting dates, corresponding to monthly vesting over five years. The shortest vesting structures are "cliff vests," where all of the options in a given grant vest on the same day. From Panel D, the industry with the largest average number of vesting dates is construction and manufacturing (SICs 1 and 2), although the median is 4.00. Transportation, communication, and utilities (SIC 4) and retail (SIC 5) have the fewest average and median number of vesting dates before the grant is fully vested. Panel E shows that the average percentage of a grant that vests on the first vesting period is 51% across all industries, the median is 33%, and the standard deviation is quite large at 36%. Again, these differences reflect the heterogeneity in vesting structures across industries.

As shown in Panel F, the average number of months from the grant to full vesting is 57 over all industries, and the median is 38 months. Retail (SIC 5) and financial services firms (SIC 6) have the shortest time to full vesting. The only homogeneous contractual feature of ESO grants across firms is the maturity in months from the issuance date to the date of expiry. Panel G shows that the term of ESOs is quite uniform at 10 years although some 15- and 4-year maturity options are granted by some firms.

Panel H of Table II presents results on a key feature of employee exercise behavior at the firms in our sample. As shown, on average only 67% of vested option positions are exercised at exercise events. The industry with the lowest fractional exercise is the computer services industry (SIC 7), which is only 57% on average, while the industries with the highest fractional exercise rate are the educational and health services industries (SICs 8 and 9), at 92%. The importance of fractional exercise across industries provides further justification for our use of a fractional-logistic estimator.

In summary, two structurally important features emerge from the stock option exercise patterns observed in our sample. First, many employees hold more than one option grant and make exercise decisions over more than one vested option at any given time. For this reason, estimation strategies must account for the correlated decision structure of employee option exercise. Second, many option positions are exercised fractionally, that is, the proportion of outstanding vested options that are exercised at exercise events can be substantially less than one. A "successful" econometric methodology must therefore account for fractional exercise behavior if it is to avoid significant misspecification bias and inaccurate option valuation. Overall, our sample comprises a representative mix of large and small firms and includes all of the employees who receive option grants within these firms.

C. Covariates

To select variables X_{ijt} to explain the conditional rate of voluntary exercise $G(X_{ijt}\beta)$ described in Section II, we draw from three strands of the literature: standard theory of value-maximizing option exercise; recent theory of utility-maximizing option exercise, which accounts for constraints on hedging options and related portfolio effects; and the empirical employee option exercise literature, which incorporates additional variables associated with grant characteristics such as vesting structure and other, possibly psychological, factors in decision making.

C.1. Standard Option Exercise Factors

According to standard theory of value-maximizing option exercise, the option holder can trade or hedge the option freely and therefore chooses to maximize the option's market value. This involves a trade-off between the benefit of postponing exercise, to put off payment of the strike price and leave the option alive, and the cost of losing dividend payments while the option remains unexercised.[12] For an ordinary American call on a stock in a Black-Scholes framework, the value-maximizing policy is described by a critical stock price boundary—above which it is optimal to exercise and below which it is optimal to continue holding the option. The critical stock price is increasing in stock return volatility, time to expiration, and the riskless rate, and decreasing in the dividend payout rate (see Cox and Rubinstein (1985), Kim (1990)). In a heterogeneous pool of option holders operating in complex real-life environments, we do not expect to see decision making as exact as a critical boundary policy implies, but to the extent that employee option exercise decisions have a value-maximizing component, we would expect exercise to be more likely the higher the stock price relative to the strike price, the higher an imminent dividend, the lower the stock return volatility, and the closer the option expiration date. We thus include the following four covariates suggested by this theory:

Price-to-strike ratio: The ratio of the split-adjusted stock price to the option strike price minus 1.

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[12] See, for example, Merton (1973), Van Moerbeke (1976), Roll (1977), Geske (1979), Whaley (1981), and Kim (1990).

Volatility: The annualized volatility of the daily stock return, estimated over the prior three months.

Dividend in next two weeks: The product of a dummy indicating whether a dividend will be paid within the next 14 calendar days and the ratio of the dividend payment to the current stock price.

Years to expiration: The number of years remaining until the expiration date of the grant.

C.2. Portfolio Factors

Unlike listed options, ESOs are explicitly nontransferable. In addition, Section 16-c of the Securities Exchange Act imposes hedging constraints by prohibiting corporate insiders from taking short positions in their companies' stock, and transaction costs and other frictions may limit the ability of other employees to short stock as well. Early exercise may thus be the only practical way for employees to diversify away their exposure to stock price risk. In this case, the exercise decision trades off the benefit of postponing payment of the strike price and leaving the option alive against the cost of bearing additional stock price risk, as well as the cost of losing dividends. The optimal exercise policy for employee options can therefore look quite different from that for standard American calls (see, as previously discussed, Huddart (1994), Marcus and Kulatilaka (1994), Carpenter (1998), Detemple and Sundaresan (1999), Ingersoll (2006), Leung and Sircar (2009), Grasselli and Henderson (2009), Carpenter, Stanton, and Wallace (2010)).[13]

Carpenter, Stanton, and Wallace (2010) model a risk-averse employee who maximizes expected utility by simultaneously choosing an exercise policy for his options and a dynamic trading strategy in the market and a riskless asset for his outside wealth. Based on a theoretical analysis of the optimal exercise boundary, they show that, as with a standard American call, exercise is more likely the greater is the dividend rate. However, unlike a standard American call:

1. exercise is more likely if the employee has greater absolute risk aversion;
2. exercise is less likely with greater employee wealth, provided the employee has decreasing absolute risk aversion;
3. exercise is decreasing in the absolute magnitude of the correlation between the stock return and the market return; and
4. the effect of greater volatility or longer time to expiration is ambiguous.

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[13] Although noninsiders are not subject to Section 16-c, existing empirical evidence suggests that early exercise of ESOs is pervasive across all ranks of employees (Huddart and Lang (1996)), perhaps because short-selling stock is costly.

This last result is due to the conflicting effects of employee risk aversion and the convexity of the option payoff.[14] Note that, in standard option theory, variables such as volatility and time to expiration only affect exercise behavior when the underlying stock pays dividends. However, in the utility maximization framework for nontransferable options, these variables can affect exercise behavior even in the absence of dividends, though the sign of the effect is theoretically ambiguous.

Following this literature, we incorporate three variables that proxy for the ease of hedging the underlying stock price risk and employee demand for hedging this risk:

Correlation: The correlation between the stock return and the return on the S&P 500 Composite Index, estimated from daily returns over the prior three months.
Black-Scholes employee option risk: The log of employee portfolio dollar volatility derived from employee option holdings, approximated as the total Black-Scholes delta of the employee's option portfolio times the dollar volatility of the stock price.[15]
Black-Scholes employee option wealth: The log of the total Black-Scholes value of the employee's option portfolio.[16]

The correlation of the underlying stock return with the market return can indicate the employee's degree of underdiversification. However, if hedging with a tradable market asset is possible, a higher correlation can indicate more scope for hedging the stock risk inherent in the option. Thus, the sign of the effect of the correlation is an open question.

The degree to which the employee's portfolio is exposed to stock risk also indicates demand for hedging through early exercise. On the other hand, the wealthier the employee, relative to a given option position, the less likely he will be to exercise the option early for diversification reasons. Our proprietary data on employee options do not include information about employee stock risk exposure and wealth derived from restricted stock, salary, and private accounts, so we proxy for employee risk and wealth based on characteristics of the employee's option portfolio. For portfolio risk, we approximate the contribution of the employee's option portfolio to the dollar volatility of his total portfolio using his total Black-Scholes option delta times the dollar volatility of the stock. If the options were European, this quantity would represent the loading of dollar portfolio value on stock risk due to options. This measure ignores additional

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[14] Both exercise behavior and option value are monotonic in volatility in models with an exogenously specified exercise boundary (see, e.g., Jennergren and Näslund (1993), Carr and Linetsky (2000), Cvitanić, Wiener, and Zapatero (2008), Hull and White (2004)).

[15] The option delta is calculated from the traditional dividend-adjusted Black-Scholes formula, evaluated using a stock-specific dividend rate and volatility, a risk-free rate of 5%, and the option-specific strike price and contractual expiration date.

[16] This is calculated using the traditional dividend-adjusted Black-Scholes option pricing formula, evaluated using the stock-specific dividend rate and volatility, a risk-free rate of 5%, and the option-specific strike price and contractual expiration date.

exposure to the stock price that the employee might have through restricted stock or human capital, but is likely to be highly correlated with the employee's overall dollar exposure to the stock price risk. Similarly, we approximate employee wealth using the Black-Scholes value of the employee's option holdings. This ignores other sources of wealth from salary, restricted stock, and private accounts, but again is likely to be highly correlated with overall wealth. These measures of portfolio risk and portfolio wealth are positively, but by no means perfectly, correlated. Since there is considerable skewness in the distributions of these measures due to the outsized positions of the large option holders, especially in technology and financial services firms, we use the logarithm of both of these measures in our estimation.

C.3. Additional Factors

In addition to factors motivated by value maximization and portfolio theory, empirical studies of exercise behavior incorporate variables motivated by contractual features of employee options and patterns in financial decision making uncovered by the behavioral literature. First, employee option grants typically vest in stages, as frequently as annually, quarterly, or even monthly. The vesting of a new tranche can represent the relaxation of a constraint, thus triggering exercise as a release of pent-up demand. In addition, even when employees already hold a portfolio of vested options, the passage of a vesting date is often accompanied by an email notification from the plan administrator, which can serve as a reminder for employees to consider exercising options. To capture the potential impact of these two effects, we include among the covariates X_{ijt} both a dummy indicating whether a vesting date for the grant has occurred in the previous two weeks and this indicator interacted with the length of time between the prior two vesting dates. Included together, these two covariates capture the magnitude of the response to the passage of the vesting date as an affine function of the time between vesting dates and leave the effect of vesting date frequency on option value an open question.

Next, a number of studies show that "reference points" affect many financial decisions. For example, Barberis and Xiong (2012) find that investors are especially willing to realize gains when the stock price hits a new high, and Huddart, Lang, and Yetman (2009) find additional volume and price patterns around a stock's 52-week high. To incorporate such reference dependence, we include an indicator for whether the stock price is in the 90th percentile of its past year's distribution.

Employee rank could also affect option exercise decisions for a variety of reasons. The behavioral literature finds that managerial overconfidence accounts for distortions in decision making, as in Malmendier and Tate (2005, 2008). High-ranking employees might also exhibit differential exercise patterns because of their portfolio composition, exposure to information flows, or political incentives, such as pressure to keep skin in the game. Our data do not systematically include information about employee rank, so our primary proxy for an employee's rank uses the total Black-Scholes value of his option holdings:

we identify high-ranking option holders at a given firm as those who are ever among the top-10 option holders at the firm. To the extent that options are granted when their performance incentives have greatest benefit, these top-10 option holders are likely to be the key decision makers at the firm. A subsample of firms includes data from human resources on employee rank. For these firms, we alternatively identify high-ranking option holders as those with the most senior titles, typically C-level.

Finally, employee gender and age could affect exercise decisions. For example, Barber and Odean (2001) find significant differences in the trading patterns of men and women, with men the more frequent traders, perhaps reflecting a difference in confidence. Employee gender could also affect option exercise decisions for rational reasons, such as gender-related differences in partner income and related household wealth. Employee age might also affect risk attitudes and planning horizons in ways that affect exercise decisions. A subsample of 59 firms provides data on employee gender and age. Of the firms that provide data on employee rank, 22 firms also provide data on employee gender and age.

In summary, we include the following additional variables in our empirical specification:

Vesting date in past two weeks: A dummy indicating whether a vesting date has occurred in the previous two weeks for the given grant.

Vesting date in past two weeks × years between prior two vesting dates: The above indicator interacted with the length of time in years between the prior two vesting dates for the given grant.

Price $\geq 90^{\text{th}}$ *percentile of prior-year distribution*: A dummy indicating whether the current stock price exceeds the 90^{th} percentile of its distribution over the prior year.

*Top-*10 *option holder*: A dummy indicating whether the employee is among the top-10 option holders at the firm in any year, ranked by total Black-Scholes value of vested option holdings.

Executive: An indicator of whether the employee is a senior executive.

Male: An indicator of whether the option holder is male.

Age: The age of the option holder.

C.4. Summary Statistics for Covariates

Table III presents summary statistics for the employee option exercise factors of interest for the full 88-firm sample, the 59-firm sample for which data on employee gender and age are available, and the 22-firm sample for which data on employee gender, age, and rank are available. In the estimation, our goal is to model voluntary option exercise, so we begin with the sample of employee-grant-days for which the option is in the money and vested. We then eliminate days that are within six months of the grant expiration date or within six months of a cancellation of any option by that employee, as most cancellations are associated with employment termination, which could force an exercise. The remaining employee-grant-days are treated as days on which the employee

Table III

Summary Statistics for Observed ESO Exercise Factors

This table presents summary statistics for the ESO exercise factors used in the fractional-logistic specifications. The summary statistics for these factors are based on their observed values on employee-grant-days on which the employee had vested in-the-money options that could be exercised. The full sample includes 88 firms. The 59-firm sample includes only those firms that reported information on gender and age. The 22-firm sample includes only those firms that reported information on gender, age, and employee rank. Price-to-strike ratio is the ratio of the split-adjusted stock price to the option strike price minus 1. Volatility is the annualized volatility of the daily stock return, estimated over the prior three months. Dividend in next two weeks is the product of a dummy indicating that a dividend will be paid within the next 14 calendar days and the ratio of the dividend payment to the current stock price. Correlation is the correlation between the stock return and the return on the S&P 500 Composite Index, estimated from daily returns over the prior three months. Black-Scholes employee option risk is the log of employee portfolio dollar volatility derived from employee option holdings, approximated as the total Black-Scholes delta of the employee's option portfolio × the dollar volatility of the stock price. Black-Scholes employee option wealth is the log of the total Black-Scholes value of the employee's option portfolio. Vesting date in past two weeks is a dummy indicating whether a vesting date has occurred in the previous two weeks for the given grant. Price $\geq 90^{\text{th}}$ percentile of prior-year distribution is a dummy indicating whether the current stock price exceeds the 90^{th} percentile of its distribution over the prior year. Top-10 option holder is a dummy indicating whether the employee is among the top-10 option holders at the firm in any year, ranked by total Black-Scholes value of vested option holdings. Executive is a dummy indicating whether the employee is a senior executive. Male is a dummy indicating whether the option holder is male. Age is the age of the option holder.

Number of Employee-Grant-Day Observations	88-Firm Sample 378,286,473		59-Firm Sample 238,690,684		22-Firm Sample 59,238,669	
	Mean	*SD*	Mean	*SD*	Mean	*SD*
Standard option exercise factors						
Price-to-strike ratio	3.46	10.68	3.38	9.16	1.34	2.46
Volatility	0.34	0.19	0.29	0.16	0.27	0.12
Dividend in next two weeks	0.0007	0.0070	0.0008	0.0029	0.0012	0.0037
Years to expiration	6.05	2.17	5.94	2.16	6.31	2.00
Portfolio factors						
Correlation	0.50	0.18	0.52	0.19	0.54	0.20
Black-Scholes employee option risk	13.60	2.88	13.35	3.10	12.10	3.03
Black-Scholes employee option wealth	12.06	2.76	11.96	2.97	10.73	2.99
Additional factors						
Vesting date in past two weeks	0.028	0.166	0.020	0.141	0.027	0.162
Vesting date in past two weeks × years between prior two vesting dates	0.016	0.152	0.015	0.168	0.016	0.157
Price $\geq 90^{\text{th}}$ percentile of prior-year distribution	0.30	0.46	0.30	0.46	0.28	0.45
Top-decile option holder/ executive	0.03	0.17	0.02	0.15	0.12	0.33
Male			0.57	0.49	0.59	0.49
Age			45.63	9.64	45.21	9.89

has a choice about whether and how many options to exercise. The full 88-firm sample provides 378 million employee-grant-days on which the employee has vested in-the-money options that could potentially be exercised voluntarily. The summary statistics for each explanatory variable are based on observed values over the 378 million employee-grant-days. The 59-firm sample provides 239 million employee-grant-day observations for each explanatory variable. The 22-firm samples provides 59 million observations.

As can be seen in Table III, the full-sample and the 59-firm-sample summary statistics are very similar. Within the class of standard option exercise factors, the price-to-strike ratios are deeply in the money on average, with a mean of 3.46 and 3.38 and *SD* of 10.68 and 9.16, respectively. Returns are slightly more volatile in the full sample, with an average volatility of 34%, than in the subsample, where average volatility is 29%. The dividend factor is measured as an indicator variable times the dividend payout rate. Many firms in the sample, especially firms in the tech industry, did not pay dividends over the analysis period. Accordingly, the summary statistics indicate a very low average incidence of dividend payment, 0.0007, for the full sample and only slightly higher incidence, 0.0008, for the 59-firm sample. The average number of years remaining to expiration is 6.05 years for the full sample and 5.94 years for the 59-firm subsample, reflecting the importance of early exercise. The summary statistics for standard factors in the 22-firm sample are generally similar to those in the larger samples, with the exception of the price-to-strike ratio, which has a lower mean, 1.34, and a lower *SD*, 2.46.

Our proxies for the portfolio factors are the correlation between the stock return and the market return, Black-Scholes employee option risk, and Black-Scholes option wealth. As Table III shows, the average correlation is near 50% in all three samples. The average log of option risk and log of option wealth measures are also similar across all samples. Table III further shows that 2% to 3% of the observed employee-grant-days occur within two weeks of a vesting event, and about 30% occur on days with price realizations greater than the 90^{th} percentile of the prior-year distribution. Moreover, about 2% to 3% of observations correspond to top-10 option holders in the two larger samples, whereas 12% of observations correspond to senior executive option holders in the smaller sample. In the samples with data on employee gender and age, 57% to 59% of observations are for male employees and the average employee age is about 45.

IV. Estimation Results

Given the importance of differentiating between the behavior of high-ranking and other employees, our baseline specification includes all of the variables described above, plus dummy variables that indicate whether the employee is of high rank. The results are reported in Table IV. There are five specifications in all. Specifications (1) and (2) show what can go wrong when we do not include the full list of variables. In particular, specification (1) includes only the standard option exercise factors, while specification (2) includes only the standard

Table IV

Baseline Estimation Results

This table presents coefficient estimates and standard errors (in parentheses) for alternative specifications of the fractional-logistic estimator. The dependent variable is the fraction of remaining vested options exercised by an employee from a given grant on a given trading day. Specification (1) uses only the standard option exercise factors as explanatory variables. Specification (2) augments these with portfolio factors. Specification (3) adds additional factors based on vesting events, prior stock price path, and employee rank. Specifications (4) and (5) add additional factors based on employee gender and age. In specifications (3) and (4), top-ranked employees are defined as top-10 option holders. In specification (5), top-ranked employees are defined as senior executives.

			Standard, Portfolio, and Additional Factors		
	Standard Factors	Standard and Portfolio Factors	Top-10 Holders		Executive
	(1)	(2)	(3)	(4)	(5)
Standard factors					
Constant	−7.5240	−7.7470	−8.8192	−7.9787	−9.9165
	(0.0102)	(0.0196)	(0.0198)	(0.0638)	(0.1262)
Price-to-strike ratio	0.0080	0.0080	0.0070	0.0080	0.0217
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0021)
Volatility	−0.3217	−0.5492	−0.2460	−0.6664	0.2005
	(0.0138)	(0.0205)	(0.0206)	(0.0304)	(0.0996)
Div next two weeks	−4.6165	−4.4559	1.2100	9.1965	7.3247
	(0.5038)	(0.4973)	(0.1668)	(0.2599)	(1.5831)
Years to expiration	0.0840	0.0855	0.0211	−0.0478	−0.3237
	(0.0011)	(0.0011)	(0.0011)	(0.0017)	(0.0047)
Portfolio factors					
Correlation		0.1855	0.4564	1.1923	1.0295
		(0.0136)	(0.0139)	(0.0194)	(0.0406)
BS Empl. option risk		0.0980	0.2727	0.2701	0.2366
		(0.0079)	(0.0071)	(0.0086)	(0.0312)
BS Empl. option wealth		−0.0936	−0.2605	−0.2536	−0.1890
		(0.0080)	(0.0072)	(0.0087)	(0.0310)
Additional factors					
Vest past two weeks			1.9964	2.5820	3.9163
			(0.0089)	(0.0091)	(0.0316)
Vest past two weeks × years bet. prior two vest dates			0.9280	0.4736	0.9577
			(0.0041)	(0.0042)	(0.0132)
Price ≥ 90th percentile of prior-year distribution			1.1691	1.0424	1.1387
			(0.0056)	(0.0069)	(0.0136)
Top-10 option holders/executive			−0.2455	−0.2325	−0.0898
			(0.0165)	(0.0229)	(0.0182)
Male				0.2089	0.0676
				(0.0072)	(0.0144)
Age				−0.0053	0.0718
				(0.0025)	(0.0052)
Age^2				−0.0000	−0.0008
				(0.0000)	(0.0001)

(*Continued*)

Table IV—*Continued*

	Standard Factors	Standard and Portfolio Factors	Standard, Portfolio, and Additional Factors		
			Top-10 Holders		Executive
	(1)	(2)	(3)	(4)	(5)
Number of firms	88	88	88	59	22
Number of employee-grant-day observations	379M	379M	379M	239M	59M

and portfolio factors. The main results are presented in specifications (3) to (5). In specifications (3) and (4), top-ranked employees are identified as top-10 option holders. Specification (3) uses the full 88-firm sample but does not include age and sex, which are not available for all 88 firms. Specification (4), in contrast, includes age and sex using the 59-firm subsample that includes information on these variables. Specification (5) is the same as specification (4) but defines top-ranked employees as those with the most senior rank and restricts attention to the 22-firm subsample with these data.[17]

Standard option exercise factors: Consistent with the predictions of most theoretical models of option exercise, the coefficient on the price-to-strike ratio is positive and highly statistically significant in all specifications, indicating that option exercise is more likely the deeper the option is in the money.

The estimated coefficients on volatility and years to expiration vary across specifications, inconsistent with standard (tradable) option pricing theory, but consistent with the result from utility maximization theory that the effects of these variables on exercise behavior are ambiguous once risk aversion is taken into account. On the other hand, theory from Carpenter, Stanton, and Wallace (2010) gives a clear prediction that a higher dividend rate should accelerate exercise. In the estimation results in Table IV, the coefficient on the dividend factor is indeed positive in our main specifications, that is, in specifications (3) to (5), but is negative in specifications (1) and (2), which only include subsets of the variables. This highlights the importance of controlling for vesting structure, stock price path, and employee characteristics in order to draw correct inferences about exercise behavior.

Portfolio factors: Among the portfolio factors, stock return correlation with the market increases the rate of option exercise. While inconsistent with the

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[17] Prior to 2005, when the FASB started requiring firms to recognize the cost of employee option compensation in their income statements, there was an incentive to use option compensation due to its favorable accounting treatment. This incentive disappeared in 2005, so it is possible that the characteristics of those compensated with stock options—and their exercise behavior—changed post-2005. Unfortunately, given the unbalanced panel structure of our data set, we do not have enough coverage across firms after 2005 for us to be able to compare the two subperiods.

predictions of Carpenter, Stanton, and Wallace (2010), which assumes that the option holder can short the market portfolio freely, the positive correlation effect makes sense if employees cannot easily hedge away their exposure to the market. In that case, the extra correlation with the market would add to total portfolio risk. The coefficients on Black-Scholes option risk and Black-Scholes option wealth are consistently of the expected sign, indicating that, as shown by Carpenter, Stanton, and Wallace (2010), employee wealth and risk aversion matter to their option exercise decisions. In particular, all else equal, employee wealth decreases the propensity to exercise, assuming decreasing absolute risk aversion, whereas undiversifiable exposure to the stock price increases it. As Hall and Murphy (2002) explain, undiversifiable exposure to the stock price creates performance incentives, but it also increases incentives to exercise options early.

Vesting structure and path-dependence: Turning to the additional factors motivated by the behavioral literature and previous empirical studies, Table IV highlights the importance of the option vesting structure. Employee exercise rates jump up in the two weeks after the passage of a vesting date. In addition, the effect of the passage of a vesting date is greater the more time has elapsed since the previous vesting date. The fact that the coefficients on both the vesting date indicator and this indicator interacted with the length of time between the prior two vesting dates are large and positive suggests that both a pent-up demand effect and a mechanical reminder effect may be at work in determining the effect of the passage of a vesting date on the exercise decision. In addition, employees exercise a significantly larger fraction of outstanding options when the stock price is greater than or equal to the 90th percentile of its distribution over the prior calendar year. These results are broadly consistent with Heath, Huddart, and Lang (1999), and are in line with the idea that employees tie their exercise decisions to cognitive benchmarks as a means of reducing monitoring costs.

Gender and age: Specifications (4) and (5) of Table IV show that male employees have a significantly greater propensity to exercise their options than female employees. This may reflect portfolio or other rational factors. As discussed in Section III.C.3, to the extent that labor force participation by males in white-collar professions is greater than participation by females, female employees may have greater wealth, or less correlated wealth, from partner income, which could slow the rate of option exercise. The result is also consistent, however, with evidence in the behavioral literature that men tend to be more active traders (see, e.g., Barber and Odean (2001)). Finally, the likelihood of fractional exercise decreases with employee age, suggesting that older employees are less likely to exercise options early. This result is more consistent with the option exercise-related effects of an accumulation of wealth with age than with the age-related U-shaped patterns found in Agarwal et al. (2009).

Employee rank: As discussed in Section III, the exercise decisions of highly ranked employees could differ fundamentally from those of lower ranked

Table V

Change in Exercise Rate for One-Standard-Deviation Increase in Exercise Factors

This table presents the percentage change in the estimated exercise rate G for one-standard-deviation increase in the exercise factors, relative to the exercise rate evaluated at the sample mean values of the exercise factors, based on specification (4) in Table IV.

Standard factors	
Price-to-strike ratio	8
Volatility	−10
Dividend in next two weeks	3
Years to expiration	−10
Portfolio factors	
Correlation	23
Employee option portfolio	11
Additional factors	
Vesting date in past two weeks	37
Vesting date in past two weeks times years between prior two vesting dates	8
Price $\geq$ 90th percentile prior-year distribution	50
Top-10 option holders/executive	−4
Male	10
Age	−5

employees for a number of reasons. Table IV reports the effects of employee rank using the top-10 option holder indicator in specifications (3) and (4) and the senior executive indicator in specification (5). In each specification, the high-rank indicator has a significantly negative coefficient, consistent with the employee rank effects reported in previous studies, based on a much smaller sample (Armstrong, Jagolinzer, and Larcker (2007)) or insiders only (Bettis, Bizjak, and Lemmon (2005), Klein and Maug (2011)). The slower exercise rates of top employees may in principle be the effect of greater outside wealth, but we control for employee wealth as proxied by total Black-Scholes option value. The slower exercise rates might also relate to better opportunities to hedge option risk, as suggested by Bettis, Bizjak, and Lemmon (2001), or managerial overconfidence, as suggested by Malmendier and Tate (2005).

Relative importance of the factors: To investigate the relative importance of each of the factors described above, Table V also reports the effect on the conditional exercise rate of increasing each factor by one sample standard deviation, starting at the sample mean of each variable.[18] Of the standard option exercise factors, the price-to-strike ratio, volatility, and time to expiration all have significant effects, as expected. However, other variables are even more important, with the two most significant variables being whether the price is

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[18] Because the two Black-Scholes variables are highly correlated with each other, we combine their effect here, showing the effect of a joint one-standard-deviation increase in both variables.

greater than the 90th percentile of prices during the prior year and whether there has been a vesting date in the past two weeks. Both of these variables are outside the set of factors that would be expected to be important either in a classical Black-Scholes setting or in a Black-Scholes setting augmented by portfolio factors.

A. Further Investigation of Employee-Rank Effects

Because the exercise decisions faced by high-ranking and lower ranking employees may be quite different, a simple rank dummy may not fully capture these differences. Table VI therefore presents the results of separate estimations for highly ranked versus lower ranked employees, using the same two alternative definitions of highly ranked employees as in Table IV. For each specification, the results are presented in three columns. The first column reports the coefficients and standard errors for lower ranked employees. The second column reports the results for top employees, and the third column reports the differences between the two coefficients, with their standard errors. Specification (6) is estimated, like specification (4), using the sample of 59 firms that provide data on age and sex. Specification (7), like specification (5), uses the senior executive indicator to designate top employees and the sample of 22 firms that provide data on employee rank, gender, and age.

As shown in Table VI, the price-to-strike ratio remains a consistently positive predictor of the exercise rate, although the difference across top-ranked and lower ranked employees varies across specifications. The dividend effect, where significant, is positive in all but one case. The correlation effect is also positive in all but one case, and tends to be stronger among lower ranked employees, suggesting that top employees might have greater ability to hedge market risk. However, the effects of Black-Scholes employee risk and wealth are more variable across samples and specifications.

The most striking and consistent effects are for the additional exercise factors. Both the vesting event indicator and the product of the vesting indicator and the length of time between prior vesting dates have large positive coefficients in all cases. Moreover, the coefficients on the simple vesting event indicator are significantly larger for lower ranked than for top employees, while the coefficients on the product of the vesting indicator and the length of time between prior vesting dates are significantly larger for top employees than for lower ranked employees. This suggests the notification effect is larger for lower ranked employees, while the pent-up demand effect is larger for top employees. The coefficient on the indicator for whether price exceeds the 90th percentile of its prior-year distribution is large and positive in each case, with the effect significantly larger for lower ranked employees than for top employees. Conversely, while the coefficient on the male employee indicator is large and positive in each case, the male effect is significantly stronger among senior executives. These results suggest that behavioral effects vary across employee ranks.

Table VI

Estimation Results for Top-Ranked and Lower Ranked Employees

This table presents coefficient estimates and standard errors (in parentheses) for two specifications of the fractional-logistic estimator incorporating rank interaction effects. The dependent variable is the fraction of remaining vested options exercised by an employee from a given grant on a given trading day. Columns (1) to (3) give estimation results for specification (6) with controls for standard, portfolio, and behavioral factors using the subsample of 59 firms that reported information on gender and age and with the interaction of the top-10 option holder indicator with each of the other explanatory variables. Columns (4) to (6) give results for specification (7), which includes controls for standard, portfolio, and behavioral factors and the interaction of the senior executive indicator with each of the other explanatory variables and uses the subsample of 22 firms that reported information on employee gender, age, and rank.

	Top-10, 59-Firm Sample 239M Observations Specification (6)			Executive, 22-Firm Sample 59M Observations Specification (7)		
	Lower	Top	Diff	Lower	Top	Diff
Standard Factors						
Constant	−8.0505	−8.3923	−0.3418	−9.6167	−9.9842	−0.3676
	(0.0650)	(0.5899)	(0.5930)	(0.1322)	(0.4731)	(0.4906)
Price-to-strike ratio	0.0082	0.0054	−0.0027	0.0169	0.0272	0.0103
	(0.0002)	(0.0005)	(0.0005)	(0.0024)	(0.0043)	(0.0049)
Volatility	−0.8511	0.3595	1.2106	1.0220	−3.2749	−4.2969
	(0.0392)	(0.1133)	(0.1198)	(0.1039)	(0.2682)	(0.2845)
Div next two weeks	10.3836	−9.5760	−19.9595	6.3412	11.2023	4.8610
	(0.2829)	(11.2650)	(11.2684)	(1.7331)	(3.8949)	(4.2387)
Years to expiration	−0.0473	−0.1130	−0.0657	−0.3645	−0.1290	0.2355
	(0.0018)	(0.0123)	(0.0128)	(0.0053)	(0.0097)	(0.0110)
Portfolio factors						
Correlation	1.1774	1.1468	−0.0306	1.1425	0.9411	−0.2014
	(0.0196)	(0.1242)	(0.1255)	(0.0442)	(0.1004)	(0.1071)
BS Empl. option risk	0.3394	−0.0697	−0.4091	−0.0698	1.5971	1.6669
	(0.0135)	(0.0321)	(0.0348)	(0.0351)	(0.0721)	(0.0802)
BS Empl. option wealth	−0.3256	0.0997	0.4253	0.1230	−1.6129	−1.7360
	(0.0139)	(0.0313)	(0.0342)	(0.0349)	(0.0732)	(0.0810)
Additional factors						
Vest past two weeks	2.5885	2.3019	−0.2867	4.1218	3.0223	−1.0995
	(0.0091)	(0.0821)	(0.0826)	(0.0359)	(0.0598)	(0.0696)
Vest past two weeks × years bet. prior two vest dates	0.4676	0.7325	0.2649	0.9018	1.2146	0.3128
	(0.0042)	(0.0394)	(0.0396)	(0.0144)	(0.0270)	(0.0306)
Price $\geq$ 90th percentile of prior-year dist'n	1.0493	0.8312	−0.2181	1.1653	1.0243	−0.1410
	(0.0070)	(0.0457)	(0.0463)	(0.0151)	(0.0308)	(0.0342)
Male	0.2108	0.1228	−0.0880	0.0213	0.2485	0.2271
	(0.0072)	(0.0889)	(0.0892)	(0.0158)	(0.0375)	(0.0407)
Age	−0.0029	0.0149	0.0178	0.0711	0.0329	−0.0382
	(0.0026)	(0.0214)	(0.0216)	(0.0055)	(0.0192)	(0.0199)
Age^2	−0.0001	−0.0001	−0.0001	−0.0008	−0.0004	0.0003
	(0.0000)	(0.0002)	(0.0002)	(0.0001)	(0.0002)	(0.0002)

V. Implications for Employee Option Cost to Firms

To evaluate the economic importance of the exercise factor effects documented above, in this section we quantify their implications for employee option cost to the firm, as modeled in equation (2) in Section II. The results reveal three striking effects. First, the degree of grading in the vesting structure has a large effect on option cost: as vesting varies from cliff to monthly, the option cost typically falls 11% to 16%. Second, employee rank effects are important: the values of options of top-ranked employees are consistently 2% to 7% above those of lower ranked employees. Third, gender effects are also significant: options granted to men are worth 2% to 4% less than the same options granted to women.

Our framework also allows us to compare the option cost implied by the estimated exercise behavior with the Black-Scholes-based approximations permitted by the Financial Accounting Standards Board(FASB) and used by most practitioners. As we explain in Section V.B, these approximations incorporate the potential for early exercise by adjusting the effective option expiration date downward in various ways. This treats all exercise as occurring on a single date, which leads to approximation errors. We show that the approximation errors can be large and vary systematically with firm characteristics. Finally, to address the limitations of these approximations in a way that fits conveniently into common practice, we develop an analytical approximation of the option's implied Black-Scholes term, which is as easy or easier to use than existing methods permitted by the FASB and matches the estimated option values much more closely.

To implement the option valuation modeled in equation (2) in Section II, we need an estimate of the employee termination rate λ as well as the voluntary exercise rate function $G(X\beta)$. It is well known that ESOs are canceled for exogenous reasons due to retirement, voluntary or involuntary resignation, or death. Recent work by Dahiya and Yermack (2008) finds that firms have diverse "sunset" policies for cancellations due to retirement or death, but quite uniform cancellation policies for resignations. In particular, they find that 85% of firms give managers three months or less to exercise their options after a resignation. Accordingly, our option valuation assumes that, upon termination, options that are vested and in the money are exercised immediately, while options that are unvested or out of the money are canceled.

All 88 of our firms report the exact date for all employee grants that are canceled. In Table VII, we report the average employee-grant termination hazard rate λ by one-digit SIC code obtained from the estimation of the employee-grant termination hazards controlling for vesting structure and the number of months that the option has been outstanding for all firms within the SIC code. As shown in the table, the overall mean hazard rate is 10.7% and the range across all of the SIC codes is between 4% for SIC 2 and 15% for SICs 8 and 9.[19]

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[19] For the subset of 46 firms that report complete data on the reason for the cancellation, we find that the average hazard of death is 0.1%, the average hazard for retirement is 1%, the average

Table VII

Summary Statistics for the Employee/Grant Cancellation Hazards by One-Digit SIC Code

Industry	One-Digit SIC Code	Mean Cancellation Hazard
Construction and Manufacturing	1 and 2	0.13
	3	0.12
Transportation, communications, and utilities	4	0.10
Retail	5	0.10
Finance, insurance, and real estate	6	0.04
Services	7	0.12
	8 and 9	0.15
Overall mean		0.107
Overall median		0.087

Based on these results, our base-case valuations assume a termination rate λ equal to 10%.

A. *Exercise Factor Effects in Employee Option Cost*

To illustrate the economic significance of the various exercise factors, Table VIII presents option values for the estimated exercise functions described in Tables IV and VI, and quantifies the factor effects on option cost across a range of parametrizations. All option values are for a 10-year at-the-money option with strike equal to $100 for a firm in SIC 3, assuming the riskless interest rate is 5% and the employee termination rate is 10%, in line with our sample average. In the base case, stock return volatility is 30%, the average stock return volatility reported by Campbell et al. (2001); the dividend rate is 0, for ease of interpretation; the stock return correlation with the market is 50%, our sample average; and the option grant vests in year 2. The underlying stock price is $100, as the vast majority of ESOs are granted at the money. The table presents option values for a range of parametrizations that vary vesting structure, volatility, and dividend rate around the base case.

Columns (1), (2), and (4) of Table VIII assume exercise according to specification (4) in Table IV with the standard, portfolio, and additional factors and top-10 holder fixed effects. The simulations set employee risk, wealth, and age equal to their sample average values conditional on rank. Columns (6), (7), and (9) assume exercise according to specification (6) in Table VI with the standard, portfolio, and additional factors and top-10 holder interaction effects, and set employee risk, wealth, and age equal to their sample average values

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hazard for voluntary resignation is 5%, and the average hazard for involuntary resignation is 1%. Dahiya and Yermack (2008) find higher frequency counts for deaths (3.15%) and retirements (4.43%) in a sample of the top-five highest paid employees in 389 S&P 500 firms. These frequencies are lower in our sample because it also includes all of the ESOs granted to lower ranked employees in each of our firms.

Table VIII

Exercise Factor Effects in Employee Option Values

This table presents option values and percentage differences based on the estimated exercise functions presented in Tables IV and VI. All option values are for a 10-year at-the-money option with strike equal to 100, assuming the riskless interest rate is 5%, the employee termination rate is 10%, and the firm is in SIC 3. The base case assumes that stock return volatility is 30% and the dividend rate is 0, the stock return correlation with the market is 50%, and the option grant vests in year 2. Columns (1), (2), and (4) use coefficient estimates from specification (4) in Table IV with the standard, portfolio, and additional factors and top-10 holder fixed effects, and set employee risk, wealth, and age equal to their sample average values conditional on rank. Columns (6), (7), and (9) use coefficient estimates from specification (6) in Table VI with the standard, portfolio, and additional factors and top-10 holder interaction effects, and set employee risk, wealth, and age equal to their sample average values conditional on rank. Rank effect is the percentage difference in ESO value between top-10 and lower ranked option holders. Gender effect is the percentage difference in ESO value between male and female option holders.

	ESO Value with Std, Ptf, and Add'l Factors with Top-10 Fixed Effects					ESO Value with Std, Ptf, and Add'l Factors with Top-10 Interaction Effects				
	Lower Male ($)	Top Male ($)	Rank Effect (%)	Top Female ($)	Gender Effect (%)	Lower Male ($)	Top Male ($)	Rank Effect (%)	Top Female ($)	Gender Effect (%)
Panel A: Vesting-Frequency Effects										
Cliff: Year 2	30.79	31.30	2	32.02	−2	30.86	31.21	1	31.64	−1
Annually: Years 1 to 4	29.89	30.37	2	31.04	−2	29.95	30.50	2	30.89	−1
Quarterly: Years 1 to 4	28.55	29.15	2	30.01	−3	28.62	29.90	4	30.37	−2
Monthly: Years 1 to 4	25.71	26.35	2	27.32	−4	25.78	27.67	7	28.24	−2
Vesting-frequency effect (%)	16	16		15		16	11		11	
Panel B: Volatility Effects										
0.20	24.64	25.18	2	25.93	−3	24.64	25.47	3	25.90	−2
0.30	30.79	31.30	2	32.02	−2	30.86	31.21	1	31.64	−1
0.40	36.82	37.33	1	38.02	−2	36.97	36.86	0	37.30	−1
0.50	42.53	43.02	1	43.68	−2	42.74	42.17	−1	42.63	−1
0.60	47.78	48.24	1	48.86	−1	48.04	47.05	−2	47.51	−1

(Continued)

Table VIII—*Continued*

	ESO Value with Std, Ptf, and Add'l Factors with Top-10 Fixed Effects					ESO Value with Std, Ptf, and Add'l Factors with Top-10 Interaction Effects				
	Lower Male ($)	Top Male ($)	Rank Effect (%)	Top Female ($)	Gender Effect (%)	Lower Male ($)	Top Male ($)	Rank Effect (%)	Top Female ($)	Gender Effect (%)
Panel C: Dividend Effects										
0.00	30.79	31.30	2	32.02	−2	30.86	31.21	1	31.64	−1
0.01	27.60	27.98	1	28.49	−2	27.65	27.92	1	28.24	−1
0.02	24.69	24.95	1	25.30	−1	24.73	24.92	1	25.14	−1
0.03	22.05	22.21	1	22.42	−1	22.07	22.20	1	22.33	−1
0.04	19.65	19.73	0	19.83	−1	19.66	19.72	0	19.79	0
0.05	17.48	17.50	0	17.50	0	17.49	17.49	0	17.50	0

conditional on rank. The columns labeled “Rank effect” show the percentage difference in ESO value between top-10 and lower ranked option holders. Those labeled “Gender effect” show the percentage difference in ESO value between male and female option holders.

Table VIII reveals three striking effects in the value of ESOs. First, the degree of grading in the vesting structure matters a great deal. Panel A shows option values for four different vesting structures, all with an average vesting date of approximately two years but with different degrees of grading. In the first row, all options vest at year 2. In the second row, one-fourth of the option grant vests in each of years 1 through 4. The third row assumes quarterly vesting in equal amounts throughout the first four years, and the last row assumes monthly vesting throughout the first four years. However, Table VIII shows that when vesting date effects in exercise are taken into account, the option cost falls as much as 16% from the case of cliff vesting to the case of monthly vesting. As the estimation results in Tables IV and VI show, the probability of an option exercise jumps up immediately after a vesting date. Although the estimation results show that more frequent vesters respond less to the passage of a vesting date, the constant term in the affine vesting-date response function is large, which leads to a pronounced vesting frequency effect in option cost. This disproportionate response may be triggered by email notifications from plan administrators that serve as a reminder to consider the option portfolio. Taking this tendency into account highlights the sensitivity of option value to vesting structure around a given average vesting period, and hence the economic importance of incorporating details of the vesting structure.

Second, employee rank effects are material. With rank modeled as a fixed effect according to specification (4) in Table IV, the cost of options held by top-10 option holders is typically 2% higher than the cost of options held by lower ranked employees. This follows from the estimation result that top-10 holders exercise options significantly more slowly than lower ranked employees. The exception is in the case of a high dividend rate, where slower exercise can reduce option value. When the employee rank indicator is interacted with other covariates according to specification (6) in Table VI, the rank effect varies more widely across parametrizations, reaching as much as 7% in the case of monthly vesting. This reflects the result that top-10 option holders react significantly less than lower ranked employees to the mere passage of a vesting date, as seen by the difference in their coefficients on the vesting indicator in Table VI.

Third, employee gender effects are also important. As the table shows, the value of options held by men is around 2% to 4% less than the value of options held by women. This reflects the tendency of men to exercise their options faster than women, all else equal. While this result might reflect in part men’s lower average partner income, it also suggests that the tendency of men to trade more frequently than women, as documented by Barber and Odean (2001), extends to the options arena, where extra trading has a clear negative impact on value. In summary, these additional, possibly behavioral, factors have a highly significant impact not only on employee option exercise rates, but also on the economic cost of these options to firms. Again, the one exception is when the dividend is very high, in which case faster exercise can add value.

B. Black-Scholes Approximations Currently Used in Practice

Since 2005, the FASB has required firms to recognize the cost of employee option compensation in their income statements. FAS 123R outlines a variety of potential methods, ranging from a Modified Black-Scholes method that uses the Black-Scholes formula with the option's actual time to expiration replaced with its expected life, to lattice and other methods. More recently, SAB 110 approves the use of the Black-Scholes formula with the option's stated maturity replaced by the average of the vesting date and stated maturity. Each approval indicates an expectation that other methods will develop as more information about option exercise behavior becomes available. However, the vast majority of firms still use one of these Black-Scholes-based approximations because of their relative simplicity. In Table IX, we compare option values derived from actual exercise behavior as estimated in Section IV with the Black-Scholes-based approximations used in practice, as well as a new method developed here. This analysis measures the approximation errors and biases in the methods permitted by the FASB, each of which allow firms to use the Black-Scholes formula with a downward-adjusted expiration date to account for early exercises. In the next section, we develop and propose a new analytic approximation with an implied option term fitted from the empirically estimated option valuation function from Section IV, which, when used as the expiration date in the Black-Scholes formula, delivers a value that much more closely approximates the true ESO value than existing methods.

Table IX reports option values under alternative valuation methods for a range of different parametrizations. As in Table VIII, the base-case parametrization assumes that the option has a contractual term of 10 years to expiration and a strike price of \$100, and that it vests at year 2. Moreover, the underlying stock price is \$100, the stock return volatility is 30%, the dividend rate is 0, the stock return correlation with the market is 50%, the riskless interest rate is 5%, and the annual employee termination rate is 10%. Each panel of the table shows the effect on the different option values of varying one of these parameters around this base case. In parametrizations with a positive dividend rate, the Black-Scholes-based approximations use the Black-Scholes formula adjusted for a continuous dividend. Panels A through D focus on at-the-money options at the grant date with 10 years to expiration. Panels E and F consider seasoned options that are closer to expiration and not necessarily at the money. Such options might be revalued by companies during an option repricing or corporate restructuring. For example, after the financial crisis of 2007 to 2008, large numbers of new options went deep out of the money. To restore incentives, some firms replaced the old out-of-the-money options with an equal present value of new at-the-money options. Similar swaps and settlements occur during corporate restructurings. For this purpose, seasoned options often need to be revalued in practice.

Table IX

Employee Option Values and Black-Scholes-Based Approximations

This table presents option values based on specification (4) in Table IV for a 52-year-old top-10 male option holder at a firm in SIC 3, labeled ESO value, and corresponding Black-Scholes approximations. BS to full term is the probability that the option vests times the Black-Scholes option value using the option's stated maturity. Midlife is the average of the option vesting date and stated maturity, and BS to midlife is the probability that the option vests times the Black-Scholes option value assuming expiration at the option's midlife. Exp term is the expected term of the option conditional on vesting, given the estimated exercise function and assumed termination rate. BS to exp term is the probability that the option vests times the Black-Scholes value of the option, assuming expiration at the option's expected term conditional on vesting. Implied term is the Black-Scholes term implied by the ESO value, conditional on vesting. Fitted term is the polynomial approximation of the implied term. BS to fitted term is the Black-Scholes value to this fitted term times the probability of vesting. The base-case option has a full term of 10 years, a strike price of $100, and vests at year 2. The base case assumes an underlying stock price of $100, 10% annual termination rate, 30% volatility, 0 dividend rate, 50% correlation with the market, 5% riskless rate, and 11% expected market return.

	ESO Value	BS to Full Term	Pct Err	Midlife	BS to Midlife	Pct Err	Exp Term	BS to Exp Term	Pct Err	Implied Term	Fitted Term	BS to Fitted Term	Pct Err
Panel A: Vesting Structure Effects													
Cliff: Year 2	31.30	43.04	37	6.00	32.63	4	5.94	32.45	4	5.57	5.57	31.28	–0
Annual: 1 to 4	30.37	41.20	36	6.25	31.95	5	6.09	31.49	4	5.71	5.69	30.32	–0
Qrt: 1 to 4	29.15	42.79	47	6.06	32.63	12	5.42	30.63	5	4.97	4.96	29.15	–0
Mth: 1 to 4	26.35	43.15	64	6.02	32.78	24	4.61	28.16	7	4.10	4.10	26.33	–0
Panel B: Volatility Effects													
0.2	25.18	37.00	47	6.00	26.83	7	5.50	25.37	1	5.43	5.57	25.57	2
0.3	31.30	43.04	37	6.00	32.63	4	5.94	32.45	4	5.57	5.57	31.28	–0
0.4	37.33	49.25	32	6.00	38.53	3	6.25	39.32	5	5.64	5.57	37.09	–1
0.5	43.02	55.11	28	6.00	44.22	3	6.48	45.80	6	5.65	5.57	42.71	–1
0.6	48.24	60.40	25	6.00	49.57	3	6.66	51.77	7	5.63	5.57	48.01	–0
Panel C: Dividend Rate Effects													
0.00	31.30	43.04	37	6.00	32.63	4	5.94	32.45	4	5.57	5.57	31.28	–0
0.01	27.98	36.58	31	6.00	28.97	4	5.99	28.94	3	5.59	5.57	27.91	–0

(Continued)

Table IX—*Continued*

	ESO Value	BS to Full Term	Pct Err	Midlife	BS to Midlife	Pct Err	Exp Term	BS to Exp Term	Pct Err	Implied Term	Fitted Term	BS to Fitted Term	Pct Err
0.02	24.95	30.94	24	6.00	25.63	3	6.04	25.70	3	5.63	5.57	24.83	−0
0.03	22.21	26.03	17	6.00	22.60	2	6.09	22.71	2	5.71	5.57	22.02	−1
0.04	19.73	21.77	10	6.00	19.86	1	6.14	19.98	1	5.85	5.57	19.46	−1
0.05	17.50	18.11	4	6.00	17.39	−1	6.20	17.49	−0	6.22	5.57	17.14	−2
Panel D: Termination-Rate Effects													
0.04	38.09	48.53	27	6.00	36.79	−3	6.78	39.37	3	6.38	6.36	38.00	−0
0.07	34.47	45.70	33	6.00	34.65	1	6.33	35.69	4	5.94	5.93	34.41	−0
0.10	31.30	43.04	37	6.00	32.63	4	5.94	32.45	4	5.57	5.57	31.28	−0
0.13	28.52	40.53	42	6.00	30.73	8	5.60	29.57	4	5.26	5.26	28.52	−0
0.16	26.07	38.17	46	6.00	28.94	11	5.31	27.01	4	4.99	4.99	26.08	0
Panel E: Time Remaining Effects													
10	31.30	43.04	37	6.00	32.63	4	5.94	32.45	4	5.57	5.57	31.28	−0
9	30.58	45.01	47	5.00	32.54	6	4.94	32.30	6	4.48	4.63	31.14	2
8	30.52	46.70	53	4.00	31.65	4	4.30	33.00	8	3.76	3.86	30.99	2
7	29.43	43.42	48	3.50	29.30	−0	4.00	31.65	8	3.53	3.63	29.91	2
6	28.15	39.86	42	3.00	26.81	−5	3.66	30.08	7	3.26	3.36	28.60	2
Panel F: Moneyness Effects (Time Remaining = Eight Years)													
50	6.47	11.77	82	4.00	4.32	−33	5.21	6.59	2	5.15	5.14	6.46	−0
75	16.65	27.51	65	4.00	15.25	−8	4.82	18.06	8	4.40	4.36	16.52	−1
100	30.52	46.70	53	4.00	31.65	4	4.30	33.00	8	3.76	3.86	30.99	2
125	49.18	67.91	38	4.00	51.51	5	3.96	51.29	4	3.54	3.71	50.04	2
150	71.36	90.34	27	4.00	73.42	3	3.85	72.68	2	3.60	3.73	72.06	1
200	118.79	137.28	16	4.00	120.36	1	3.78	119.29	0	3.68	3.82	119.51	1

The first column of Table IX, labeled "ESO value," is the option value, that is, its cost to the firm, based on specification (4) of Table IV with the standard, portfolio, and additional factors for a 52-year-old top-10 male at a firm in SIC 3. The second column of Table IX, labeled "BS to full term," is the probability that the option vests under the assumed termination rate times the Black-Scholes option value using the option's stated expiration date. As is well known, the traditional Black-Scholes value using the option's stated expiration date massively overstates option value, except in the case of a high dividend rate, because it does not incorporate any early exercise. The base-case error is 37% and ranges as high as 82%.

The next three columns illustrate the valuation method permitted in SAB 110. This adjusts the option's expiration date down to the average of the option vesting date and stated expiration date, a simple adjustment for the presence of early exercise that is straightforward for firms to apply without having to collect and analyze data on exercise behavior. The column labeled "Midlife" shows the average of the option vesting date and stated maturity, and the column labeled "BS to midlife" shows the probability that the option vests times the Black-Scholes option value assuming expiration at the option's midlife. In the case of graded vesting, the midlife is averaged across tranches. As the table shows, this method is a straightforward and significant improvement over the traditional Black-Scholes formula, and certainly an improvement over the pre-2005 practice of expensing zero option cost for at-the-money options. However, the errors can still be quite large because the adjustment to the option expiration date fails to capture the actual exercise patterns observed for employee options and their variation across stock price paths and other aspects of the economic setting. The base-case error is 4%, but ranges as high as 24% with graded vesting structures, under which exercise occurs at a much higher rate. The errors are also large in the cases of low volatility, low dividend rate, and for some of the seasoned options, where the simple adjustment of Black-Scholes to midlife falls well short of incorporating actual early exercise patterns.

The next three columns illustrate the Modified Black-Scholes method permitted by FAS 123R, which replaces the option's stated expiration date in the Black-Scholes formula with an estimate of its expected term. Though in principle the Modified Black-Scholes method could capture some of the exercise policy effects through adjustments to the option expected term, it suffers from two major problems. The first is that the estimation of the option's expected term is as difficult as estimating the option cost itself. This is because the realized option term depends on the stock price path as well as any other variables that affect exercise decisions, and no single firm is likely to have a sufficiently long history of option outcomes to perform this estimation with any precision. For example, two identical firms could grant options with the same expected term, but if one firm estimates option term from a past history with poor stock price performance and lots of long-lived options that finished near or out of the money, while the other estimates option term from a past history with good stock price performance and past options exercised early, the first firm can end up expensing its new option grants at a much greater cost than the second firm because of this normal small-sample estimation error. The second issue is that, even without estimation error in the option expected term, the Modified Black-Scholes approximation error can vary widely and systematically with firm characteristics because even the exact expected option term does not sufficiently summarize the exercise policy.

To make this second point, we compute the Modified Black-Scholes option approximations using the exact expected terms corresponding to exercise policies underlying the ESO values in the first column. These ESO values assume that the option holder follows the exercise policy estimated in specification (4) of Table IV and are simulated using equation (2) in Section II. To get the corresponding Modified Black-Scholes approximations, we simulate the option's true expected term also assuming that the option holder follows the exercise policy estimated in specification (4) of Table IV. The option expected term is with respect to the true probability measure, consistent with firms' practice of using sample means of realized option lives, so it depends on the true expected return on the stock. We compute this expected term conditional on vesting, and then multiply the Black-Scholes option value with expiration date set equal to this conditional expected term by the probability of vesting to arrive at the final Modified Black-Scholes cost approximation. Formally, the option's true expected term conditional on vesting is

$$L_t = E_t\left\{\int_{t\vee t_v}^{T} \tau\,(G_\tau + \lambda)e^{-\int_{t\vee t_v}^{\tau}(G_s+\lambda)\,ds}\,d\tau + T\,e^{-\int_{t\vee t_v}^{T}(G_s+\lambda)\,ds}\right\}, \tag{5}$$

where the stock price follows the process

$$dS/S = (\mu - \delta)\,dt + \sigma\,dZ. \tag{6}$$

We assume an expected market risk premium of 6% and a mean stock return commensurate with its correlation with the market under the Capital Asset Pricing Model(CAPM).

In theory, the Modified Black-Scholes approximation can either under- or overstate the true option value, depending on the exercise policy. To see why, consider two special cases, and for simplicity assume immediate vesting. First, if the option holder follows the value-maximizing exercise policy in the presence of dividends, as in standard theory, then the true option value will be greater than the Black-Scholes value to any deterministic expiration date, so it will exceed the Modified Black-Scholes approximation. Alternatively, suppose the option is stopped, either through exercise or cancellation, at a purely exogenous rate independent of the stock price. Then the option value is the average Black-Scholes value over possible stopping dates, while the Modified Black-Scholes approximation is the Black-Scholes value to the average stopping date, so by Jensen's inequality the Modified Black-Scholes approximation will overstate the true value, since the Black-Scholes value tends to be concave in the option expiration date. The exercise policies followed in practice contain elements of both of these cases, so the Modified Black-Scholes approximation can in principle either over- or understate the true ESO cost, as the examples in Table IX show.

The column labeled "Exp term" in Table IX shows the true expected option term, conditional on vesting, based on the assumed employee exercise rate function and termination rate underlying the valuations in the first column labeled "ESO value." The column labeled "BS to exp term" is the probability that the option vests times the Black-Scholes value of the option, assuming expiration at the option's expected term conditional on vesting. As the table shows, even in the hypothetically ideal case in which the expected option term is estimated perfectly, the pricing errors can be significant. The exact expected term captures some features of the real exercise policy, so it generally does better than the simple midlife adjustment in the absence of estimation error. However, even in this ideal case, the errors can be fairly large. Moreover, the errors vary systematically with firm characteristics.

As Panels B and C of Table IX show, the Modified Black-Scholes method tends to overstate option value at high-volatility firms and understate it at high-dividend firms. In Panel B, the true ESO value increases more slowly than the Modified Black-Scholes value as stock return volatility goes up. The intuition for this is that, with higher volatility, the lognormal distribution of the stock price becomes more skewed and the probability that the option ends up out of the money goes up, as does the option's expected term, which increases the Modified Black-Scholes value, although these out-of-the-money stock price paths do not actually contribute to greater option value. At the same time, under the estimated exercise policy, the exercise rate increases as the option gets deeper in the money, so high stock price paths are likely to be ones with earlier exercises, which the Modified Black-Scholes value fails to capture, and this early elimination of options along potentially high stock price paths becomes more costly relative to exercise at the average term the higher the volatility of the stock price.

In Panel C of Table IX, ESO value falls more slowly than the Modified Black-Scholes value as the dividend payout rate increases. This is because the value-maximizing exercise policy for a call on a dividend-paying stock calls for exercising the option early, once the stock price rises sufficiently high, and the estimated exercise policy has an element of this feature. In other words, for high-dividend-paying stocks, the Modified Black-Scholes value assumes that the option is exercised too late along high stock price paths, and so it understates the true option value, which incorporates earlier exercise and greater dividend capture along these paths.

More generally, as this Modified Black-Scholes approximation is based on a deterministic exercise date, it fails to fully capture the effects of exercise policies that are dynamic functions of the stock price and other variables. When the additional challenge of estimating the expected option term from a firm's historical data is taken into account, it is not clear that this method dominates the SAB 110 midlife method. Many firms do not have these data, and even those that do probably face significant sampling error because they only observe exercises along a single stock price path.

C. A New Black-Scholes-Based Analytic Approximation

Although Monte Carlo simulation methods are relatively standard in the quantitative finance industry, the vast majority of firms use one of the closed-form Black-Scholes approximations permitted by FAS 123R or SAB 110 for employee option valuation because of their simplicity. In addition, the empirical corporate finance literature often uses Black-Scholes methods to value executive stock options. For a recent example, see Shue and Townsend (2017). To facilitate the use of our valuation method by firms, auditors, and researchers, in this section we develop a closed-form approximation to the option's "implied" term, the term that, when used as the expiration date in the Black-Scholes formula, yields the correct option value according to our estimated exercise policy.

Good overviews of approximation theory for functions that are expensive or otherwise difficult to evaluate directly include Hamming (1973), Powell (1981), and Judd (1998, chapter 6). Polynomial approximations are commonly used, motivated by considering either Taylor series or the Weierstrass theorem (see Judd (1998, theorem 6.5.1)), which tells us that any continuous function on an interval $[a, b]$ can be uniformly approximated by a polynomial.[20] Applications in finance include Longstaff and Schwartz (2001), who estimate the continuation value of an option in a Monte Carlo framework by regressing discounted payoffs along each path on polynomial functions of the underlying variables. Similar techniques are also used in the neural-network literature (see, e.g., Barron (1993), Refenes, Burgess, and Bentz (1997)).

We regress the implied maturity for a set of ESOs on a set of polynomial basis functions, similar to Longstaff and Schwartz (2001). Since we are using polynomials, the basis functions could, in principle, be simple powers of the variables $1, x, x^2, \ldots$ and their cross-products. However, the high correlation between these terms often leads to serious numerical problems when evaluating the regression coefficients (see Powell (1981, chapter 11)). To avoid these problems, it is common to use instead as basis functions a family of *orthogonal polynomials*, which satisfy the relationship

$$\int_a^b P_i(x)P_j(x)w(x)\,dx = 0, \tag{7}$$

where P_i and P_j are polynomials of (different) orders i and j, and $w(x)$ is a specified *weighting function*. There are several commonly used families of orthogonal polynomials, each corresponding to a different weighting function. For example, setting $[a, b] = [-1, 1]$ and $w(x) = \frac{1}{\sqrt{1-x^2}}$ in equation (7) leads to the *Chebyshev polynomials*, defined (see Powell (1981, pp. 38–39)) as

$$T_n(x) = \cos\left(n\cos^{-1} x\right).$$

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[20] Stone's theorem (see Judd (1998, theorem 6.12.1)) allows the Weierstrass theorem to be extended to multiple dimensions.

To calculate our approximation, we regress implied maturities against products of Chebyshev polynomials in the following six variables:

(1) the log of the total number of vesting dates;
(2) the last vesting date;
(3) the exogenous termination rate;
(4) the log of the ratio of stock price to exercise price, $\log(S/K)$;
(5) the time remaining to option expiration; and
(6) the constant in the exercise function.

We use 456 basis functions altogether, the number of products of polynomials in these variables where each individual polynomial is of degree no higher than four, and where the total degree of all polynomials in the product does not exceed five. There are 15,625 different values in the regression, each corresponding to a different combination of the six variables.

The last four columns of Table IX show the Black-Scholes terms implied by the true ESO value conditional on vesting, the fitted value of this implied term approximated by the polynomials described above, the Black-Scholes value with this fitted term times the probability of vesting, and the percentage error relative to the true ESO value. The Black-Scholes value calculated with the fitted implied term is significantly closer to the true value than either of the two other Black-Scholes-based approximations, typically within 2%. By contrast, column (6) of Table IX shows that the percentage pricing errors of the midlife method range into the double digits with graded vesting. Even the ideal Modified Black-Scholes value, with the hypothetically exact expected term, easily gives errors on the order of 5%, and the approximation errors produced in practice, using firms' sample average realized option terms, are potentially much larger. Panels B and C show that the fitted Black-Scholes errors are very small across a wide range of values for stock return volatility and dividend rate, even though the fitted term has not been constructed to vary with these variables because the Black-Scholes formula incorporates much of the effects of variation in volatility and dividend rate through their effects on the underlying stock price process. Panel D shows that this new approximation method works extremely well across a wide range of termination rates.

Panels E and F show option values and approximation errors for seasoned options with less than 10 years remaining to expiration and options that are in or out of the money. Firms may need to revalue such options in the case of a corporate or option grant restructuring. Here again, the Black-Scholes formula applied to the fitted term approximates the value of these options much more closely than the methods used in current practice.

Our analytic approximation based on a fitted implied term and corresponding Black-Scholes value represents a significant improvement over existing methods. It approximates option value much more closely across a wide range of scenarios and is as easy or easier to calculate. Further, it requires no additional data or estimation, as in the expected term method, because our method already correctly incorporates the information in exercise data from grants to over 200,000 employees across 59 firms. And while calculating the fitted term requires summation of 456 different polynomials, this can be easily done in a spreadsheet. This fitted Black-Scholes-based analytic approximation can serve as the basis for a new employee valuation method for accounting and other valuation purposes in practice, as well as in corporate finance research on executive and employee compensation.

VI. Conclusion

In this paper, we develop a methodology for estimating option exercise as a function of the stock price path, time to expiration, and firm and option holder characteristics. Our estimation is based on a fractional-logistic approach, which accounts for correlation between exercises by the same employee and provides an estimated exercise function that can serve as an input to option valuation. Valuation proceeds by using the estimated exercise function to describe the option's expected payoff along each stock price path, and then computing the present value of the payoff. The estimation of empirical exercise rates also allows us to test the predictions of theoretical models of option exercise behavior.

We apply our estimation technique to a comprehensive sample of option grant and exercise data for over 290,000 employees at 88 publicly traded firms over the period 1981 to 2009. We consider the effects on exercise behavior of firm, contract, and option holder characteristics drawn from standard option theory, portfolio theory, and the empirical employee option exercise literature. We find new exercise factor effects in option cost. First, vesting structure matters a lot—option cost drops 11% to 16% from cliff to monthly vesting since the passage of vesting dates triggers early exercises. Moreover, top-ranked option holders exercise more slowly, with their options typically worth 2% to 7% more than those of lower ranked option holders, while men exercise faster than women, reducing their options' value by around 2% to 4%. In addition, consistent with the portfolio theory literature, we find that, everything else equal, employees with more wealth tend to exercise later and employees with greater option risk tend to exercise earlier.

Finally, our results indicate that the Black-Scholes approximations used in practice can lead to significant pricing errors. Recognizing the practical appeal of such an approximation, we develop an analytical approximation of the option's implied Black-Scholes expiration date, which when incorporated into the Black-Scholes formula delivers values that much more closely approximate executive and employee option values across a wide range of parameters. Moreover, this method is much easier to use than the existing Modified Black-Scholes method because it requires no further estimation of exercise behavior or expected term.

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Appendix A: Simulation Details

As explained in Section II, in a grant that vests in fractions $\alpha_1, \ldots, \alpha_n$ at times $t_1, \ldots, t_n$, the average option value is

$$O_t = \sum_{k=1}^{n} \alpha_k \mathrm{E}_t^* \left\{ \int_{t \vee t_k}^{T} e^{-r(\tau - t)} (S_\tau - K)^+ (G_\tau + \lambda) e^{-\int_t^\tau (G_s + \lambda)\, ds}\, d\tau \right.$$
$$\left. + e^{-r(T-t)} (S_T - K)^+ e^{-\int_t^T (G_s + \lambda)\, ds} \right\}.$$

To compute O_t for a given exercise function, G, and termination rate, λ, we use Monte Carlo simulation as follows:

(1) Simulate a large number, N, of stock price paths between date 0 and option expiration date T at discrete time intervals of length Δt. For a given path i, we therefore simulate $J = T/\Delta t$ values, where the j^{th} value is calculated as

$$S^i_{(j+1)\,\Delta t} = S^i_{j\,\Delta t} \times e^{\left(r - \frac{1}{2}\sigma^2\right)\Delta t + \sigma\sqrt{\Delta t}\,\epsilon^i_j},$$

and where the ϵ^i_j are i.i.d. standard normal random variables.

(2) Along each path i, for each period j (corresponding to calendar date τ), calculate the conditional probability of the option being stopped this period, $(G_\tau + \lambda)\,\Delta t$, and the probability that the option has survived to τ, $e^{-\int_t^\tau (G_s + \lambda)\, ds}$.

(3) For each period, calculate the expected discounted cash flow conditional on the stock price path and stopping at time τ times the probability of stopping at time τ,

$$e^{-r(\tau - t)} (S_\tau - K)^+ (G_\tau + \lambda) e^{-\int_t^\tau (G_s + \lambda)\, ds}\, \Delta t,$$

and add these values for each period.

(4) Also add in the expected cash flow at expiration (date T),

$$e^{-r(T-t)} (S_T - K)^+ e^{-\int_t^T (G_s + \lambda)\, ds}.$$

(5) Repeat this for a large number of stock price paths, and average the results for each path to obtain an estimate of the option value, O_t.

To increase precision, we also use two "variance reduction" techniques in generating the paths, namely, antithetic variates and importance sampling (see Glasserman (2003)).

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