

# Payment System Externalities\*

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## Abstract

We examine how the payment processing role of banks affects their lending activity. In our model, banks operate in separate zones, and issue claims to entrepreneurs who purchase some inputs outside their own zone. Settling bank claims across zones incurs a cost. In equilibrium, a liquidity externality arises when zones are sufficiently different in their outsourcing propensities—a bank may restrict its own lending because it needs to hold liquidity against claims issued by another bank. Our work highlights that the disparate motives for interbank borrowing (investing in productive projects and managing liquidity) can have different effects on efficiency.

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# 1 Introduction

There are large payment flows between banks. In the U.S. in 2017, the Federal Funds market had an average daily volume of U.S. \$75 to \$100 billion, while CHIPS, the largest private clearing house in the US, processes transfers of \$1.5 trillion per day.<sup>1</sup> What is the role of banks in the payment system, and how does this role affect their lending function?

To address this question, we construct a stylized model with three important elements. First, banks process payments which generate interbank obligations. As in practice, claims issued on accounts at one bank may be deposited and withdrawn at another bank. Second, payment flows are tied to the level of demand deposits held by banks. Some demand deposits originate from household savings; however, banks also create demand deposits when they issue loans. Third, banks are strategic and take these flows into account. To capture the role of the payment system in ordinary or normal times, our model has no uncertainty (and hence no bank insolvency) and identical productive opportunities across banks.

Our first main result is that a liquidity externality exists across banks in normal times. Because banks are connected through the payment system, claims on a bank  $A$  can be cashed in at another bank,  $B$ . Clearing and settlement are asynchronous in our model, which requires bank  $B$  to hold liquid assets against the claims of bank  $A$  in the short term. Bank  $B$  thus loses some control over its own liquidity, and, as a consequence, restricts the quantity of loans it makes. The bank thus incurs a real cost.

In our framework, banks face equally productive investment opportunities, so the first-best outcome requires equal investment across banks. However, in equilibrium, if the flow of claims is not symmetric across banks, investment is distorted away from the first-best. Suppose the claims of bank  $A$  are more likely to be redeemed at bank  $B$  than vice versa. Then, bank  $A$  lends more than the first-best level, and bank  $B$  correspondingly cuts back on loans; that is, there is cross-sectional redistribution of economic activity. Interestingly, we find that in many situations, the liquidity externality leads to bank  $B$  borrowing from bank  $A$  in the interbank market, even though, relative to the first-best level, bank  $B$  has decreased (and bank  $A$  has increased) the loans it is making to the real sector. Our paper therefore highlights that interbank borrowing is not driven solely by differences in productivity, but can also arise from differences in spending patterns that generate differences in liquidity needs across banks.

Both in our model and in practice, interbank settlement is costly. Many payment systems require banks to post collateral, typically proportional to payment outflows, or to prefund expected

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<sup>1</sup>See <https://apps.newyorkfed.org/markets/autorates/fed%20funds> and <https://www.theclearinghouse.org/payment-systems/chips>.

outflows.<sup>2</sup> We assume in our model that net interbank claims incur a settlement cost that is borne by a bank if it is a net payer. In equilibrium, the settlement cost dampens the liquidity externality because each bank is reluctant to be a net payer at the eventual settlement date. Of course, total welfare is reduced by the additional settlement cost.

The settlement cost is not part of the first-best calculus. In the second-best outcome, the planner too is exposed to this cost, and so has an incentive to ensure that bank payment flows are symmetric. Therefore, per dollar of investment, if bank *A* is more likely to generate claims that are cleared by bank *B* than vice versa, the planner reduces lending by bank *A* and increases lending by bank *B*. Thus, the second-best planning solution yields a very different distribution of real investment than the market equilibrium.

Our model features two banks that each operate in their own local zone. Each bank makes loans to a local continuum of entrepreneurs by issuing claims, or “fountain pen money.”<sup>3</sup> That is, the quantity of lending may exceed the amount of physical currency deposited at the bank. Entrepreneurs use these claims to purchase inputs from households for their production process. Importantly, some inputs must be outsourced; that is, purchased outside the zone in which the lending bank operates. We assume the outsourcing propensity varies across the two zones.

The need for outsourcing has two consequences. First, households holding claims on a bank in another zone may cash them in at their own local bank. This interim demand for liquidity both generates a need for interbank loans and in equilibrium constrains the amount of claims banks can issue. The interest rate in the interbank loan market adjusts to clear the market, with banks acting as price-takers in this market. Second, at the end of the game, there is a need to settle up across different zones. After netting out claims, one bank may need to transfer cash to the other bank. At this point, consistent with practice, the net payer incurs a settlement cost.

There are two basic frictions in the model. First, interim liquidity needs of consumers, in conjunction with strategic behavior by banks, constrain bank lending. Second, banks face a deadweight cost to being a net payer in the payment system. This cost comprises both the collateral cost of using the current systems and the internal opportunity cost of liquidity management. The cost associated with being a net payer reduces a bank’s willingness to issue claims.

The timing of the model, with liquidity needs arising before final settlement, reflects the asynchronous nature of clearing and settlement. With many interbank transactions, settlement occurs with a lag. Sometimes, the lag is short, and may be just intra-day. In other cases, such as with banker’s acceptances or lines of credit, the lag to settlement is longer. This timing of payments

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<sup>2</sup>The Bank of International Settlements Committee on Payments and Market Infrastructures provides a comprehensive guide to payment systems in different countries (see <https://www.bis.org/cpmi/paysysinfo.htm?m=3%7C16%7C30>).

<sup>3</sup>This (somewhat dismissive) phrase is due to Tobin (1963).

matters to banks—as Afonso and Shin (2011) point out, banks generally hold low amounts of cash and reserves, and rely on the inflows from other banks to fund the the bulk of their own payment outflows.

There has been much discussion among central banks and practitioners on modernizing the payment system and reducing the cost to participants. One innovation currently being considered is Central Bank Digital Currency (CBDC), discussed in Section 4 of our paper. If wholesale CBDC reduces the settlement cost across banks, our model implies that although it increases welfare overall, it exacerbates the liquidity externality. The result is that inequality in lending across zones is increased rather than decreased. To the extent that a regulator is concerned with regional equity, this is a stark result.

Historically, the banking literature on banks has analyzed frictions related to insolvencies and to asymmetric information. Instead of considering such extreme outcomes, in this paper, we abstract away from such frictions and focus on the day-to-day operations of solvent banks, in their dual role as payment processors and lenders to the real economy. In this context, we present a new liquidity externality that is a part of normal bank operations.

Our analysis differs from the seminal work of Diamond and Dybvig (1983) in two fundamental ways. First, as we are interested in frictions other than bank insolvency, our banks face no project risk, in the sense that patient consumers in our model do not have the option to withdraw their deposits at the intermediate date. Second, in the Diamond and Dybvig world, banks are simply a conduit to channel savings from households to the real economy. We allow banks to issue fountain pen money, which implies that banks must actively choose the quantity of loans they make. In the latter vein, Donaldson, Piacentino and Thakor (2018) consider the features of banks that make them the optimal entity to issue claims, and Gu, et al. (2013) show that in a model with limited commitment, banks may emerge as trustworthy agents, with claims on bank deposits being used as means of payment.

Our paper is most closely related to the literature on liquidity management in the banking system. We build on the insights provided by Freixas, Parigi, and Rochet (2000), who model an interbank payments system based on depositors having to travel to distant places to consume. Their paper focuses on the role of lines of credit in enhancing the resiliency of the banking system when a bank can be insolvent, as such contracts allow exposed banks to spread the losses to other banks. We motivate the need for interbank payments by having some producers travel to another location. More importantly, the main insight from our model is that the commitment inherent in an interbank line of credit (which commit a bank to honoring claims issued by another bank) restricts lending *ex ante*.

Kahn and Roberds (2009) provide an introduction to the economics of payment and settlement

systems in the modern economy, and highlight the role of informational frictions. Bianchi and Bigio (2018) present a calibrated general equilibrium model in which banks receive deposits, make loans and settle reserves in the interbank market. They focus on the importance of liquidity management in the presence of credit demand and other shocks. By contrast, we focus on the effect of strategic credit supply in the absence of uncertainty or informational frictions.

The costs and benefits of gross versus net settlement are examined by, among others, Kahn and Roberds (1998). Bech and Garratt (2003) consider strategic issues in intra-day liquidity management in a real-time gross settlement system. Our time horizon is somewhat longer as we consider the period over which banks issue loans to the real economy. Thus, we limit attention to net settlement in our model.

Empirically, Ashcraft, McAndrews and Skeie (2009) present a detailed analysis of the interbank market, and show that large banks typically borrow from small banks and do so as a result of liquidity shocks that arise because of large value transfers. Craig and Ma (2018) show that, in the German interbank market, some banks are persistent borrowers and others are persistent lenders, consistent with our model.

## 2 Model

Consider an economy with two suppliers of liquid claims: a non-strategic central bank and strategic commercial banks. The central bank issues fiat money, while commercial banks issue private money that we refer to as “bank claims.” Entrepreneurs and households constitute the real part of the economy. Each entrepreneur and household is infinitesimal, and has to use bank claims to secure real goods for production or consumption.

The main interactions in our model occur between two banks. Each bank acts as a monopolist in its own segmented market of entrepreneurs and households. Such segmentation could arise from either geographic differentiation or product differentiation along an unspecified dimension. Associated with a bank is a continuum of entrepreneurs and a continuum of households, with each having mass 1. We refer to a bank and its system of entrepreneurs and households as a zone. To justify banks behaving as price-takers in the interbank loan market, we assume the economy consists of a large number  $N$  of regions, or matched pairs of banks.

Each entrepreneur owns a technology which produces output  $f(k)$  from real inputs  $k$ . We assume that  $f(k)$  is strictly increasing and strictly concave, and satisfies the Inada conditions  $\lim_{k \rightarrow 0} f'(k) = \infty$  and  $\lim_{k \rightarrow \infty} f'(k) = 0$ . The production function  $f(k)$  is the same across all entrepreneurs and all zones. The entrepreneur is penniless, and to fund investment must borrow from its affiliated bank. Bank claims are denoted by  $b$ , and are issued to entrepreneurs as demand

deposits. They correspond to Tobin’s (1963) “fountain-pen money.” The entrepreneurs use these claims to purchase inputs from households.

Following Freixas, Parigi and Rochet (2000), we introduce the need for an inter-zonal payment system by assuming that entrepreneurs must buy some inputs in the matched zone. The two zones are differentiated by the extent of outsourcing required by the technology. The outsourcing propensity is designated by  $\alpha \in \{\alpha_\ell, \alpha_h\}$ , where  $0 \leq \alpha_\ell < \alpha_h \leq 1$ . We refer to a zone with outsourcing propensity  $\alpha_i$  as zone  $i$ , and to the bank in this zone as bank  $i$ . The entrepreneur in zone  $i$  needs a quantity  $(1 - \alpha_i)k_i$  of inputs from their own zone and a quantity  $\alpha_i k_i$  of inputs from zone  $-i$ . For example, if  $\alpha_\ell = 0$ , entrepreneurs in zone  $\ell$  only purchase local inputs. Similarly, if  $\alpha_h = 1$ , the entrepreneurs in zone  $h$  purchase all their inputs from zone  $\ell$ . All inputs are supplied by households.

Households in a zone initially hold fiat currency, which they deposit into their local bank. We assume that the aggregate quantity of fiat money in each zone is the same,  $C$ . Entrepreneurs use bank claims to pay households for the inputs purchased. When entrepreneurs in zone  $h$  (say) purchase inputs from zone  $\ell$ , households in the latter zone obtain claims issued by bank  $h$ . These households deposit the claims into their own local bank,  $\ell$ . A proportion  $\lambda$  of households suffer a short-term liquidity shock that requires a cash withdrawal before output has been produced. Overall, the purchasing process thus results in the claims of a bank with type  $\alpha_i$  being held by a bank with type  $\alpha_{-i}$  and vice versa, creating the need for an inter-zonal payment system.

In addition to issuing claims, banks maintain reserve accounts at the central bank. Following practice in the U.S., in our model cash, bank claims, and central bank reserves are all denominated in the same units (dollars) and are all exchangeable at par. We assume that a safe asset (or, equivalently, a storage technology) is available that allows a bank to transfer cash or reserves across any two points of time. There is no danger of bank insolvency, and the safe rate of return is normalized to zero, so there is no need to discount bank claims. Therefore, each unit of bank claims  $b$  is priced at \$1. There is one real good that is both the input and the output, and its price is normalized to \$1. The implicit assumption here is that the input supply curve is flat at the price of \$1; assuming an upward sloping supply curve would complicate the analysis, but not qualitatively change the results.

There are three dates,  $t = 0, 1, 2$ . The timeline is presented in Figure 1. At time  $t = 0$ , local households deposit cash  $C > 0$  in their bank.<sup>4</sup> The quantity  $C$  is taken as exogenous. Each bank  $i$  then makes loans in the form of take-it-or-leave-it offers to its local entrepreneurs. It does so by creating demand deposits  $b_i$ . Typically, the loans extended, or “inside money,” exceed the

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<sup>4</sup>In keeping with standard practice, vault cash is included in the bank’s reserves. Thus, once a bank has this cash, it can now lend some of it to the other bank using reserves.

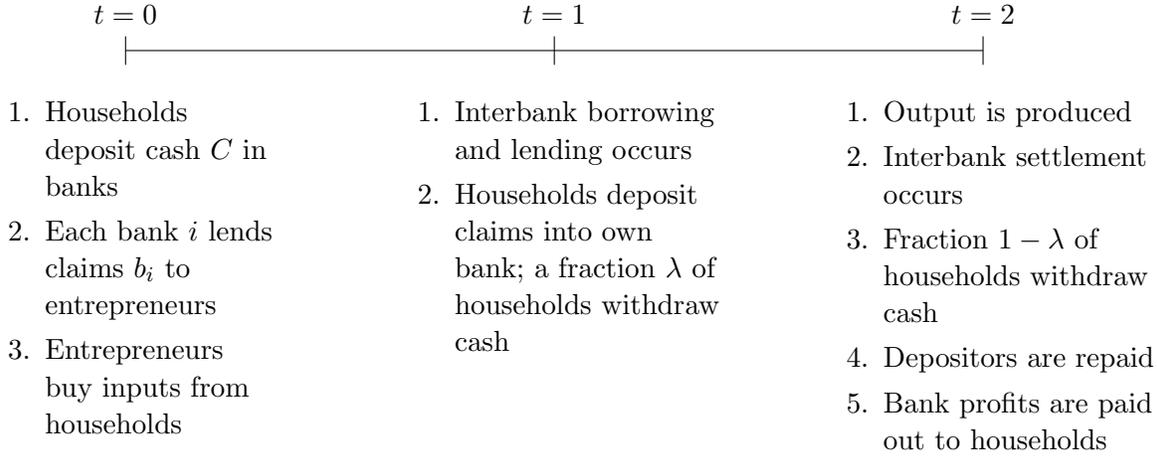


Figure 1: **Sequence of events.**

cash deposits — as discussed later, entrepreneurs’ projects can generate a surplus, creating the resources necessary to repay the higher loan amounts. Also at  $t = 0$ , entrepreneurs use bank claims to purchase inputs from households. Let  $b_i$  denote the quantity of bank claims available to entrepreneur  $i$ , and  $k_i$  the quantity of inputs she purchases from households.

At  $t = 1$ , the interbank market is active, and banks can exchange reserves with each other. This is the market for interim liquidity. We denote the interest rate (i.e., the price of borrowing and lending reserves) at which the market clears as  $r$ . We assume that the number of regions  $N$  is large, so banks act as price-takers in this market.

In addition, at  $t = 1$ , a proportion  $\lambda \in [0, 1]$  of households realize a need for consumption before the output is realized. These households arrive at their bank and immediately liquidate all their bank claims (that is, claims from both bank  $i$  and bank  $-i$ ), and also withdraw the cash they had deposited at date 0. Each bank  $i$  therefore needs to either hold enough cash (obtained at date 0 from households) or borrow reserves from the other bank to satisfy this demand. A proportion  $1 - \lambda$  of households are patient, and do not consume at this interim date.

At  $t = 2$ , production is realized, final interbank settlements are made, and all remaining value is remitted to households. When the output is produced, entrepreneurs deposit it back into their own bank, and the bank makes all required payments. Here, the consumption good is the same as the input good, and we assume it can be costlessly converted into reserves. If  $\alpha_\ell \neq \alpha_h$ , interbank settlement is required at this date. Suppose, for example, that  $\alpha_h = 1$  and  $\alpha_\ell = 0$ . In this case, all claims issued by both banks have been turned in by households to bank  $\ell$ . The claims issued by bank  $h$  remain liabilities of bank  $h$ , so bank  $h$  must transfer reserves to bank  $\ell$  to fulfill these claims. Further, if bank  $h$  had borrowed reserves of \$1 from bank  $-i$  at date 1, it now owes bank  $\ell$

a total of  $1+r$ , which also requires interbank settlement. Also at date 2, final remittances are made to households. A proportion  $1-\lambda$  of households convert their bank claims into cash at this date. The final payment to households comprises the initial cash deposits as well as all profits generated by the bank.

Observe that there are two kinds of interbank financial flows in our model. First, interbank loans are taken out at date 1 and repaid at date 2. We assume that the cost of making such transfers is zero. Second, there are flows related to the real economy, which are generated when households in zone  $-i$  obtain claims issued by bank  $i$ . We assume that it is costly for a bank to settle such claims. Specifically, a bank that in net terms is sending funds to another bank at date 2 to settle consumer payments incurs a cost  $\tau > 0$  per unit of funds transmitted. We discuss the interpretation of this settlement cost  $\tau$  in Section 2.1.

Under autarky, a bank has total funds of  $C$  available to pay out to impatient households at date 2. Suppose the bank issues a quantity  $b$  of bank claims. As a proportion  $\lambda$  of households are impatient, the total withdrawals at the interim date are  $\lambda(C+b)$ . Therefore, to satisfy the interim liquidity requirement at date 1, it must be that  $\lambda(C+b) \leq C$ , or  $b \leq \frac{(1-\lambda)C}{\lambda}$ . Denote  $I = \frac{(1-\lambda)C}{\lambda}$  as the investable funds in each zone, that is, the maximal investment a bank can make under autarky. Across a pair of matched banks in a region, the total investable funds are then  $2I$ .

Define  $\beta_i = \frac{\alpha_i}{\alpha_\ell + \alpha_h} 2I$ . Observe that,  $\alpha_\ell \beta_h = \alpha_h \beta_\ell$ , so that if bank  $h$  were to issue claims in the quantity  $\beta_\ell$ , and bank  $\ell$  in the quantity  $\beta_h$ , there would be no net settlement transfer at date 2. Thus  $\beta_h$  and  $\beta_\ell$  correspond to the investment levels attained if total investable funds across the two zones are divided in inverse proportion to their respective outsourcing propensities  $\alpha_h$  and  $\alpha_\ell$ . Because  $\alpha_h > \alpha_\ell$ , we have  $\beta_\ell < I < \beta_h$ .

We impose two parameter restrictions. The first is a restriction on the marginal productivity of the technology in the economy. The second restriction ensures that the cost of interbank transfers through the payment system,  $\tau$ , is not prohibitively large.

**Assumption 1** *The economy satisfies:*

- (i) *Sufficiently high productivity:*  $f'(\beta_h) > 1$ .
- (ii) *Small transaction costs:*  $\tau < \min \left\{ f'(\beta_h) - 1, -f''(\beta_\ell) \frac{\beta_\ell}{\alpha_\ell + \alpha_h} \right\}$ .

Part (i) of the assumption ensures that in any equilibrium, all investable funds are invested in real projects rather than in the storage technology. Part (ii) is invoked in the proof of Proposition 3 to ensure that, when  $\alpha_h$  is sufficiently high compared to  $\alpha_\ell$ , there exists an equilibrium in which bank  $h$  makes a settlement payment to bank  $\ell$ .

## 2.1 Discussion: Interpretation of Model Parameters

### *Settlement cost ( $\tau$ )*

In our model, for analytical convenience, we have final settlement at  $t = 2$  and inter-bank loans being made at  $t = 1$ . In practice, both kinds of flows occur daily, and the uncertainty associated with payment streams is larger than that associated with inter-bank loans. The cost  $\tau$  captures the opportunity cost of maintaining extra liquidity or collateral to hedge against the additional uncertainty.

In practice, payment system costs are significantly higher for banks when settling consumer payments than for purely interbank transactions. As Furfine (2011) notes, banks active in the payment system send and receive payments that are 30 times larger than their reserves. Consumer payments are settled throughout the day, so that the associated collateral requirements and especially the net debit caps bite during active trading hours for banks. To facilitate this large volume of payments, the central bank provides intra-day credit to each bank. These positions are monitored per minute, and the costs are based on net payments. Thus, as in our model, the costs are incurred by net payers.

In contrast, as mentioned by Afonso and Lagos (2014), activity in the federal funds market occurs mostly in the last two hours of the operating day (i.e., 16:30 to 18:30 eastern standard time). The loans in this market are unsecured and typically overnight, and collateral required by the payment system is returned to banks at the end of the day. Further, to the extent that some banks are persistent borrowers and others are persistent lenders in this market, the net flows are relatively small. Thus, we assume that the cost of transferring these funds is zeros, so that the overall cost of transacting in the interbank market is just the interest rate ( $r$  in our model).

There are both explicit and implicit costs associated with managing intra-day liquidity. For example, FedWire in the U.S., operated by the Federal Reserve, and CHIPS, a private clearing system, both have explicit (albeit small) fees that must be paid for using the system. More importantly, there are collateral costs as well as limits on payments. For example, FedWire requires collateral on daylight debits (i.e., net outflows), with both a fee of 50 to 150 basis points (annualized) on uncollateralized daylight overdrafts and a maximum limit on daylight overdrafts.<sup>5</sup> Of course, banks may choose to collateralize all overdrafts, but this comes at the opportunity cost of daylight capital. Daylight uses of suitable collateral include obtaining secured financing and meeting margins on derivatives positions. Similarly, CHIPS has a daily pre-funding obligation based on expected net outflows. More broadly, under the Liquidity Monitoring tools suggested by the Basel Committee on Banking Supervision, banks are required to manage their intra-day liquidity

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<sup>5</sup>See [https://www.federalreserve.gov/paymentsystems/psr\\_overview.htm](https://www.federalreserve.gov/paymentsystems/psr_overview.htm), and the references therein.

to satisfy both normal payments and stressed demands.<sup>6</sup> An industry study by the consulting firm Oliver Wyman estimates that the cost of holding liquidity reserves (both overnight and daylight) to be on the order of 100 basis points.<sup>7</sup>

#### *Proportion of impatient households ( $\lambda$ )*

In the model, a proportion  $\lambda$  of households consume at date 1, and the remaining proportion  $1 - \lambda$  consume at date 2. While we treat this proportion as exogenous, in practice it can be affected by households' choices over inter-temporal consumption. For example, the prospect of inflation between times 1 and 2 may induce households to shift consumption toward time 1, thus increasing  $\lambda$ . Similarly, security innovations that increase trust in the banking system may induce households to leave their claims in until time 2, thus reducing  $\lambda$ . This proportion can also be affected by differential consumption or transaction taxes at the two points of time.

#### *Quantity of household savings deposited at time 0 ( $C$ )*

The parameter  $C$  represents the level of household savings deposited with banks at time 0. This quantity will depend on household wealth, the level of financial inclusion in the economy, and trust in the banking system. In addition, it will be higher when it is easier for consumers to process transactions with counterparties, for example, through bank-based ACH payments and debit or credit cards. On the converse side, strict know-your-customer or KYC rules may drive some people who value privacy to alternate means of payment, leading to a reduction in  $C$ .

## **2.2 Bank's Objective Function**

The bank seeks to maximize its profit. Profit is denominated in units of the consumption good at date 2, and has three components. First, the surplus from production, which is captured entirely by the bank. Each bank is a monopolist in its own zone, and so entrepreneurs are held down to their reservation utility, which is normalized to zero. Suppose bank  $i$  issues a quantity  $b_i$  of bank claims. As claims are converted at a price of 1 into real inputs, the quantity of inputs purchased,  $k_i$ , equals  $b_i$ . Therefore, the expected surplus from production is  $f(k_i) - k_i = f(b_i) - b_i$ .

The second component of the profit function is due to interest payments in the interbank market. Let  $z_i$  denote the amount borrowed by bank  $i$  in the interbank market at date 1. Here,  $z_i < 0$  indicates that the bank is a lender rather than a borrower. The cost of borrowing in this market is  $rz_i$ , where the interest rate  $r$  is determined in equilibrium, and is taken by bank  $i$  as given.

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<sup>6</sup>See Basel Committee on Banking Supervision, "Monitoring tools for intraday liquidity management," available at <https://www.bis.org/publ/bcbs248.pdf>.

<sup>7</sup>See the Oliver Wyman Report "Intraday Liquidity," by S. Sawjany, J. Sooklal, and M.M. Lee, available at <https://www.oliverwyman.com/content/dam/oliver-wyman/v2/publications/2018/june/Intraday%20Liquidity%20Final%20Report.pdf>.

In addition, bank  $i$  may incur a settlement cost to settle claims with bank  $-i$  at date 2. Recall that entrepreneurs in zone  $i$  have purchased a quantity  $\alpha_i b_i$  of inputs in zone  $-i$ . Thus, the quantity of claims of bank  $i$  that are held by bank  $-i$  is  $\alpha_i b_i$ . Similarly,  $\alpha_{-i} b_{-i}$  represents the claims of bank  $-i$  that are held by bank  $i$ . Thus, the net payment between the banks as a result of such settlement is  $\alpha_i b_i - \alpha_{-i} b_{-i}$ . We refer to this amount as a net transfer from bank  $i$  to bank  $-i$ . If the net transfer from bank  $i$  is positive, bank  $i$  incurs an additional per unit cost of  $\tau$  to transmit it through the payment system. This is the third component of the payoff function.

Finally, note that as the return on the storage technology is zero, any amount the bank invests in this technology makes no contribution to the net profit of the bank. Putting all this together, bank  $i$ 's payoff function is

$$\pi_i = f(b_i) - b_i - r z_i - \tau \max\{\alpha_i b_i - \alpha_{-i} b_{-i}, 0\}. \quad (1)$$

The bank faces an interim solvency constraint at date 1: It must have enough cash or reserves on hand to meet household needs at that date. Households in zone  $i$  obtain  $(1 - \alpha_i) b_i$  claims from local entrepreneurs and  $\alpha_{-i} b_{-i}$  claims from distant entrepreneurs. Recall that a proportion  $\lambda$  of households cash in their bank claims at date 1. These households also withdraw the cash they had deposited into the bank at date 0. The demand for interim liquidity at bank  $i$  therefore amounts to  $\lambda(C + (1 - \alpha_i) b_i + \alpha_{-i} b_{-i})$ .

Liquidity is available to bank  $i$  from two sources: cash,  $C$ , and interbank borrowing or lending,  $z_i$ . The interim liquidity constraint for a bank at date 2 may be written as  $z_i \geq \lambda(C + (1 - \alpha_i) b_i + \alpha_{-i} b_{-i}) - C$ , or equivalently:

$$z_i \geq \lambda((1 - \alpha_i) b_i + \alpha_{-i} b_{-i} - I), \quad (2)$$

where  $I = \frac{(1-\lambda)C}{\lambda}$  represents investable funds under autarky. Any excess borrowing  $z_i > \lambda((1 - \alpha_i) b_i + \alpha_{-i} b_{-i} - I)$  is invested in the safe asset from date 1 to date 2. The bank chooses  $b_i$  and  $z_i$  to maximize its payoff  $\pi_i$  subject to the interim liquidity constraint.

The interbank rate  $r$  is an important equilibrium quantity, and is determined by market-clearing in the market for reserves at date 1. Let  $z_{ij}$  denote the net borrowing of bank  $i$  ( $i = h, \ell$ ) in region  $j$ . Summing across regions, the market-clearing constraint is:

$$\sum_{j=1}^N (z_{hj} + z_{\ell j}) = 0. \quad (3)$$

### 2.3 Planning Outcome

To establish a benchmark, we begin by considering the allocation of claims (and hence of investment and production) that would be chosen by a central planner seeking to maximize aggregate output

in the economy. As each pair of matched zones is identical, we describe the planner's problem in terms of one such pair. The outcome from a single pair is then replicated across the other  $N - 1$  pairs. The planner therefore maximizes  $\pi_h + \pi_\ell$ , the joint profit across the two zones  $h$  and  $\ell$ .

We consider two versions of the planner's problem. In both versions, the planner is subject to interim liquidity requirements at each bank. In our first-best case, the planner can costlessly make transfers to settle bank-issued claims at date 2; that is, the planner is not subject to the settlement cost  $\tau$ . The planner's first-best problem is:

$$\max_{\{b_h, b_\ell, z_h, z_\ell\}} f(b_h) + f(b_\ell) - b_h - b_\ell \quad (4)$$

$$\text{subject to :} \quad z_h + z_\ell = 0 \quad (5)$$

$$z_h \geq \lambda \left( (1 - \alpha_h) b_h + \alpha_\ell b_\ell - I \right) \quad (6)$$

$$z_\ell \geq \lambda \left( (1 - \alpha_\ell) b_\ell + \alpha_h b_h - I \right) \quad (7)$$

The planner's allocation is constrained by the need to maintain interim liquidity at each of the banks, to satisfy the liquidity demand by impatient households. We show in Lemma 1 below that the liquidity constraints (6) and (7) bind at the optimum. The market-clearing constraint may then be written as  $b_h + b_\ell = 2I$ , which is easily interpretable as a resource constraint, with  $2I$  being interpreted as the total investable funds across the two banks.

It is immediate from the concavity of the production function that the solution to the first-best problem involves each of banks  $h$  and  $\ell$  issuing the same number of claims. Let the superscript  $f$  denote the solution to the first-best problem.

**Lemma 1** *In the planner's solution to the first-best problem, the liquidity constraints bind, and  $b_h^f = b_\ell^f = I$ .*

In the second-best case, in addition to the liquidity constraints, the planner is also subject to the settlement cost for bank claims,  $\tau$ . The objective function for the second-best problem is thus:

$$\max_{\{b_h, b_\ell, z_h, z_\ell\}} f(b_h) + f(b_\ell) - b_h - b_\ell - \tau |\alpha_h b_h - \alpha_\ell b_\ell|. \quad (8)$$

The constraints are the same as in the first-best problem.

When  $\tau > 0$ , in the second-best problem the planner distorts investment toward the low-outsourcing zone, to try and avoid the settlement costs at date 2. Recall that  $\beta_i = \frac{\alpha_i}{\alpha_\ell + \alpha_h} 2I$ , and define  $\bar{\tau} = \frac{f'(\beta_\ell) - f'(\beta_h)}{\alpha_h + \alpha_\ell}$ . When  $\alpha_\ell > 0$ , we have  $\bar{\tau} \rightarrow 0$  as  $\alpha_h \rightarrow \alpha_\ell$ .

**Proposition 1** *In the solution to the second-best planning problem, the liquidity constraints bind. Further,*

(a) If  $\tau < \bar{\tau}$ :

(i) The solution satisfies  $f'(b_h^s) - f'(b_\ell^s) = \tau(\alpha_h + \alpha_\ell)$ . In particular,  $\beta_h > b_\ell^s > b_h^s > \beta_\ell$ , with  $b_\ell^s$  increasing and  $b_h^s$  decreasing in each of  $\tau, \alpha_\ell, \alpha_h$ .

(ii) Bank  $h$  makes a net transfer to bank  $\ell$  at time 2; that is,  $\alpha_h b_h^s > \alpha_\ell b_\ell^s$ .

(b) If  $\tau \geq \bar{\tau}$ , the solution has  $b_\ell^s = \beta_h > b_h^s = \beta_\ell$ . Here,  $\alpha_h b_h^s = \alpha_\ell b_\ell^s$ , so there are no net transfers at time 2.

In both cases in the proposition, the planner skews investment toward the low-outsourcing zone. If it persisted with the first-best solution of  $b_\ell^f = b_h^f = I$ , at date 2 it would be subject to a total settlement cost  $\tau(\alpha_h - \alpha_\ell)I$ . Reducing this cost requires reducing  $b_h^s$  and increasing  $b_\ell^s$ . Relative to the first-best outcome, total output is therefore lower. The tradeoff between these two costs determines the solution to the second-best problem.

When the per unit settlement cost  $\tau$  is high (part (b) of the proposition), the planner sets  $\alpha_h b_h^s = \alpha_\ell b_\ell^s$  to ensure that no overall cost for payments is incurred at date 2. If the settlement cost is lower than  $\bar{\tau}$ , as in part (a) of the proposition, the planner adjusts investments in the two zones until the marginal benefit from the saving on the total settlement cost exactly equals the marginal cost of distorting investment away from the first-best levels.

### 3 Market Equilibrium

We characterize a Nash equilibrium in the banks' lending game, with each bank taking the interest rate in the interbank market,  $r$ , as given. Our implicit assumption is that  $N$  is large, so that each zone is small relative to the size of the economy. We restrict attention to equilibria that are symmetric across regions; that is, equilibria in which each high-outsourcing bank issues the same number of claims  $b_h^*$  regardless of region, and similarly for each low-outsourcing bank. We refer to this as a region-symmetric equilibrium.

**Definition 1** *A region-symmetric market equilibrium in the model consists of claims issued by high- and low-outsourcing banks,  $b_h^*$  and  $b_\ell^*$ , net borrowing by each bank,  $z_h^*$  and  $z_\ell^*$ , and an interest rate in the interbank market,  $r^*$ , such that:*

(i) *The interim liquidity constraint of each bank  $i$ , equation (2), is satisfied.*

(ii) *For each bank  $i$ ,  $b_i^*$  and  $z_i^*$  maximize its payoff  $\pi_i$  as shown in equation (1), given the interbank interest rate,  $r^*$ , and the claims issued by its matched bank,  $b_{-i}^*$ .*

(iii) *The interbank loan market clears; that is,  $z_h^* + z_\ell^* = 0$ .*

### 3.1 Bank's Best Response

In any equilibrium, the interbank interest rate  $r^*$  will be strictly positive. To see this, add the net borrowing of the two banks,  $z_h + z_\ell$ , and impose the market-clearing constraint in the interbank loan market. This yields  $b_h + b_\ell \leq 2I$ . Therefore, for the loan market to clear, at least one bank must issue claims  $b_i \leq I$ . From Assumption 1 part (i), if bank  $i$  issued a claim quantity  $\beta_h > I$ , it would still be the case that the marginal return from lending exceeds the return on the safe asset. Therefore, to induce a bank to lend weakly less than  $I$ , the interest rate  $r^*$  must be strictly positive.<sup>8</sup>

Therefore, to determine a bank's best response, we focus on the case that the equilibrium rate in the interbank loan market is strictly positive. Observe that at any strictly positive interest rate, the liquidity constraint for each bank  $i$  will bind; that is,  $z_i = \lambda((1 - \alpha_i)b_i + \alpha_{-i}b_{-i} - I)$ .

At date 0, each bank  $i$  chooses  $b_i$ , the number of claims it issues, taking as given the claims issued by bank  $-i$  and the interest rate in the interbank market at date 1. As entrepreneurs are all identical, each entrepreneur in zone  $i$  receives the same number of claims. Recall that there is a mass 1 of entrepreneurs; we can therefore refer to  $b_i$  as the number of claims received by a single entrepreneur in zone  $i$ . As the input price is normalized to 1, we can equivalently think of  $b_i$  as the size of the real investment made by entrepreneurs in zone  $i$ .

Define  $g(x) = f'^{-1}(x)$ . That is,  $g(\cdot)$  recovers the real input level that generates a particular level of marginal product. We note that the concavity of  $f(\cdot)$  implies that  $g(x)$  is decreasing in  $x$ ; that is, higher marginal products are generated by lower input levels.

Two thresholds are useful in exhibiting the best response function of bank  $i$ . Define

$$b_i^+(r | \cdot) = g(1 + r\lambda(1 - \alpha_i)) \quad (9)$$

$$b_i^-(r | \cdot) = g(1 + r\lambda(1 - \alpha_i) + \tau\alpha_i). \quad (10)$$

Here,  $b_i^+$  denotes the investment or input level at which the marginal product in zone  $i$  is equal to  $1 + r\lambda(1 - \alpha_i)$ . Similarly,  $b_i^-$  is the investment level at which the marginal product in zone  $i$  is equal to  $1 + r\lambda(1 - \alpha_i) + \tau\alpha_i$ . As  $g(\cdot)$  is decreasing, it is immediate that when  $\tau > 0$  we have  $b_i^+ > b_i^-$ . We show that a bank which is a net receiver in the interbank payment system at date 2 will issue a quantity of claims equal to  $b_i^+$ , and correspondingly a net payer will issue claims in the amount  $b_i^-$ .

The best response of bank  $i$  depends on the claims issued by bank  $-i$  in two ways. First, bank  $i$  must ensure that it has sufficient liquidity at date 1 to meet liquidity demands by local households that hold some claims from entrepreneurs in zone  $-i$ . Second, if  $\tau > 0$  and bank  $i$  is required to make a transfer to bank  $-i$  at date 2, it incurs a transaction cost. The size of this cost depends on

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<sup>8</sup>In this argument, the specific quantity  $\beta_h$  plays a role only to the extent that it strictly exceeds  $I$ .

the relative values of  $\alpha_i b_i$  and  $\alpha_{-i} b_{-i}$ . Taking both factors into account, bank  $i$ 's best response is as follows.

**Lemma 2** *Suppose  $r > 0$ , and bank  $-i$  has issued claims  $b_{-i}$ . The best response of bank  $i$  is:*

$$b_i^* = \begin{cases} b_i^- & \text{if } b_i^- \geq \frac{\alpha_{-i}}{\alpha_i} b_{-i} \\ \frac{\alpha_{-i}}{\alpha_i} b_{-i} & \text{if } \frac{\alpha_{-i}}{\alpha_i} b_{-i} \in (b_i^-, b_i^+) \\ b_i^+ & \text{if } b_i^+ \leq \frac{\alpha_{-i}}{\alpha_i} b_{-i} \end{cases} \quad (11)$$

$$z_i^* = \lambda((1 - \alpha_i)b_i^* + \alpha_{-i}b_{-i} - I). \quad (12)$$

Observe that the net borrowing in the interbank market,  $z_i^*$ , is chosen to satisfy the bank's liquidity constraint given the claims issued by both banks,  $b_i^*$  and  $b_{-i}$ . Thus, the bank's optimal response when  $r > 0$  can be reduced to the choosing the quantity of claims to issue to local entrepreneurs.

### 3.2 Liquidity Externality

As mentioned earlier, Assumption 1 ensures that the equilibrium interbank interest rate  $r^*$  is strictly positive. Therefore, the market-clearing constraint  $z_h^* + z_\ell^* = 0$  can equivalently be written as  $b_h^* + b_\ell^* = 2I$ . That is, in any market equilibrium, total investment is equal to the total investable funds across the two zones.

To highlight the liquidity externality generated by heterogeneous outsourcing propensities, we consider a special case of our model with  $\tau = 0$ ; that is, one in which there are no settlement costs. In this setting, the first- and second-best planning outcomes coincide, and have  $b_\ell = b_h$ . However, the market equilibrium outcome features  $b_h^* > b_\ell^*$ , so that in equilibrium production is distorted toward the high-outsourcing zone. When bank  $h$  issues a greater quantity of claims, a relatively large proportion of the claims are transferred to households in zone  $\ell$ . Therefore, bank  $\ell$  must hold a greater quantity of liquid funds at date 1 to meet the short-term liquidity needs of its clients. This externality in turn inhibits the issuance of claims by bank  $\ell$ .

**Proposition 2** *Suppose that  $\tau = 0$ , so that there is no settlement cost at date 2. Then, in the market equilibrium:*

- (i) *There is a unique interest rate  $r^*$ .*
- (ii) *Bank  $h$  issues more claims than bank  $\ell$ ; that is,  $b_h^* > I > b_\ell^*$ .*
- (iii) *Bank  $h$  makes a net transfer to bank  $\ell$  at date 2; that is,  $\alpha_h b_h^* > \alpha_\ell b_\ell^*$ .*

(iv) If, in addition, either: (a)  $f(k) = A \ln k$ , where  $A > 0$ , or (b)  $\alpha_h \geq \frac{1}{2}$ , then  $z_h < 0 < z_\ell$ ; that is, in the interbank market at date 1, bank  $\ell$  borrows from bank  $h$ .

Thus, even though the technologies in the two zones have the same productivity, in a market equilibrium there can be a large dispersion in economic activity across the zones. The externality arises because aggressive lending by bank  $h$  leads to higher claims deposited at bank  $\ell$  at the interim date. This tightens the liquidity constraint of bank  $\ell$ , and inhibits its own lending. In this way, our model highlights the distinction between clearing (which occurs when bank  $\ell$  pays out on behalf of bank  $h$ ) and settlement (which occurs when bank  $h$  settles up with bank  $\ell$  at the end of the game). This distinction is common with many banking instruments. For example, it is a key part of intra-day liquidity management for banks. In foreign trade, banker's acceptances are commonly cleared in a foreign country. Alternatively, one could think of bank  $h$  maintaining a line of credit with bank  $\ell$  and drawing it down when bank  $\ell$  clears a claim it has issued.<sup>9</sup>

One way to measure the extent of the liquidity externality is to consider the gap between  $b_h^*$  and  $b_\ell^*$ . It is straightforward to show that this gap increases in the difference between  $\alpha_h$  and  $\alpha_\ell$ . Further, note that the equilibrium entails a net transfer from bank  $h$  to bank  $\ell$  at date 2; that is,  $\alpha_h b_h^* > \alpha_\ell b_\ell^*$ . When  $\tau = 0$ , there is no additional cost to such a transfer.

As bank  $h$  issues more claims than bank  $\ell$ , one may expect it is a borrower in the interbank market at date 1. Surprisingly, as part (iv) of the Proposition shows, in some cases the converse occurs, and bank  $h$  lends to bank  $\ell$  in the interbank market at date 1. In other cases, if  $\alpha_h < \frac{1}{2}$ , bank  $h$  may be either a borrower or a lender in the interbank market. For example, suppose the production function is the power function  $f(k) = Ak^y$ , where  $A > 0$  and  $y \in (0, 1)$ , and set  $\alpha_\ell = 0$ . We find in numerical examples that if resources in the economy are relatively abundant (that is, given other parameters,  $I$  is sufficiently high), then for all values of  $\alpha_h$ , bank  $h$  is the lender in the interbank market. However, if resources are scarce (i.e.,  $I$  is sufficiently low, or conversely,  $A$  is sufficiently high for a fixed value of  $I$ ), for a region of  $\alpha_h$  including zero, bank  $\ell$  is the lender in the interbank market, whereas for a region of  $\alpha_h$  including  $\frac{1}{2}$ , bank  $h$  is the lender.

In our model, the only purpose of interbank borrowing is to ensure that a bank has enough liquidity to pay off impatient households at date 3. Consider, for example, a situation with a high value of  $\alpha_h$ , above  $\frac{1}{2}$ . All else equal, if bank  $h$  expands its lending in this scenario, bank  $\ell$  needs to hold more liquidity at date 3 than bank  $h$ . Thus, bank  $h$  lends cash and reserves to bank  $\ell$ , rather than the other way around. That is, the bank that lends more to the real sector also lends more to the financial sector.

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<sup>9</sup>Allen and Gale (2000) and Freixas, Parigi, and Rochet (2000) highlight the importance of lines of credit in helping banks manage idiosyncratic shocks. Our model points to a cost of interbank credit lines, resulting from the liquidity externality.

It is important to note that, to isolate the liquidity externality, we have assumed that the productivity of each zone is the same. If differential productivity were the only source of heterogeneity in the model, then, in equilibrium, bank  $h$  both issues more claims than bank  $\ell$  and borrows from bank  $\ell$  in the interbank market. This is the more conventional model of the interbank channel, in which it serves as a device to reallocate resources to more productive banks.

More broadly, therefore, we make the point that borrowing on the interbank market has at least two purposes: First, to fund investments, which leads more productive banks to be borrowers. Second, to fund short-term liquidity needs; these needs may not be correlated with investment productivity. In the stark case of our model, productivity is the same, but the bank that lends less to entrepreneurs has a greater need for short-term liquidity—liquidity to honor the claims of the other bank. Our model therefore suggests that to understand borrowing and lending in the interbank market, it is important to distinguish between these different motivations. In particular, bank liquidity can be induced through a liquidity externality or can be intrinsically demanded due to access to better investment opportunities.

### 3.3 Equilibrium When $\tau > 0$

When the settlement cost  $\tau$  is strictly positive, the market equilibrium may be one of two kinds. In a no-payments equilibrium, neither bank makes a settlement transfer to the other bank at date 2. That is, such an equilibrium satisfies  $\alpha_\ell b_\ell^* = \alpha_h b_h^*$ , or  $b_h^* = \frac{\alpha_\ell}{\alpha_h} b_\ell^*$ . The second kind of equilibrium is a payments equilibrium in which  $\alpha_\ell b_\ell^* \neq \alpha_h b_h^*$ . In this equilibrium, there is a net settlement transfer between banks at date 2.

Intuitively, a no-payments equilibrium exists if the outsourcing propensity  $\alpha_h$  is relatively small. Recall from Proposition 2 that when  $\tau = 0$ , bank  $h$  (the high-outsourcing bank) issues more claims than bank  $\ell$ . However, if  $\tau > 0$ , bank  $h$  effectively suffers a penalty when it does so due to the settlement cost at date 2. If  $\alpha_h$  is small enough, it prefers to issue just enough claims to ensure there are no settlement payments at date 2. As the next lemma shows, in equilibrium bank  $h$  in fact issues fewer claims than bank  $\ell$  in this case.

**Lemma 3** *Suppose that  $\tau > 0$ . Then, there exists an  $\bar{\alpha} \in (\alpha_\ell, 1)$  such that, if  $\alpha_h < \bar{\alpha}$ , then in the market equilibrium  $b_h^* = \beta_\ell$  and  $b_\ell^* = \beta_h$ , and there are no transfers between banks at date 2. A range of interbank interest rates  $r^* \in [\underline{r}, \bar{r}]$  supports this equilibrium, where  $\underline{r} = \frac{f'(\beta_\ell) - 1 - \tau \alpha_h}{\lambda(1 - \alpha_h)}$  and  $\bar{r} = \frac{f'(\beta_h) - 1}{\lambda(1 - \alpha_\ell)}$ .*

Recall that  $\beta_s = \frac{\alpha_s}{\alpha_h + \alpha_\ell} 2I$  for each  $s = h, \ell$ . Thus,  $\alpha_h \beta_\ell = \alpha_\ell \beta_h$ , so that if bank  $h$  issues claims in the quantity  $\beta_\ell$  and bank  $\ell$  in the quantity  $\beta_h$ , there is no net settlement payment across the two banks at date 2. In Lemma 3, the lower threshold  $\underline{r}$  is chosen to ensure that  $b_h^- = \beta_\ell$  when  $r = \underline{r}$ ,

and the upper threshold  $\bar{r}$  is chosen to ensure that  $b_\ell^+ = \beta_h$  when  $r = \bar{r}$ . As shown in the proof of the lemma, the threshold value  $\bar{\alpha}$  is defined by the value of  $\alpha_h$  at which  $\underline{r} = \bar{r}$ . For small values of  $\tau$ , the threshold  $\bar{\alpha}$  is very close to  $\alpha_\ell$ , as shown in Example 1 in Section 4.

In practice, payment flows among banks are large. Thus, while Lemma 3 is useful in terms of analyzing our model for all parameter values, the obverse case with  $\alpha_h > \bar{\alpha}$  is of greater interest to us. When  $\alpha_h > \bar{\alpha}$ , the benefit to bank  $h$  of issuing additional claims outweighs the settlement cost it must incur at date 2, and a payments equilibrium results.

**Proposition 3** *Suppose that  $\tau > 0$  and  $\alpha_h > \bar{\alpha}$ . Then, in the market equilibrium:*

(i) *The equilibrium interbank interest rate  $r^*$  is unique.*

(ii)  *$b_h^* = b_h^- > b_h^s$  and  $b_\ell^* = b_\ell^+ < b_\ell^s$ .*

(iii) *Bank  $h$  makes a net transfer to bank  $\ell$  at date 2; that is,  $\alpha_h b_h^* > \alpha_\ell b_\ell^*$ .*

(iv) *There exists a threshold outsourcing propensity  $\hat{\alpha} \in (\bar{\alpha}, 1)$  such that, if  $\alpha < \hat{\alpha}$ , bank  $h$  issues fewer claims than bank  $\ell$  (i.e.,  $b_h^* < b_\ell^*$ ), and if  $\alpha > \hat{\alpha}$ , bank  $h$  issues more claims than bank  $\ell$  (i.e.,  $b_h^* > b_\ell^*$ ).*

(v) *If, in addition, either: (a)  $f(k) = A \ln k$ , where  $A > 0$ , or (b)  $\alpha_h \geq \max\{\frac{1}{2}, \hat{\alpha}\}$ , then  $z_h < 0 < z_\ell$ ; that is, in the interbank market at date 1, bank  $\ell$  borrows from bank  $h$ .*

Notice that, between Lemma 3 and Proposition 3, we have established that the claims issued by each bank in a market equilibrium are unique for all parameter values. In the payments equilibrium of Proposition 3, the interbank interest rate is also unique in equilibrium, whereas in the no-payments equilibrium of Lemma 3, a range of interbank interest rates supports the equilibrium. For the rest of the paper we focus on the case in which a payments equilibrium exists.

Part (ii) of the proposition shows that the equilibrium outcome is distorted away from the second-best outcome shown in Proposition 2. In particular, bank  $h$ , which has the higher outsourcing propensity, issues a greater quantity of loans to entrepreneurs than in the second-best outcome. Bank  $h$  essentially exploits the externality that when it issues claims, a greater proportion of them are cashed in at bank  $\ell$ , and it does not need to hold interim liquidity against these claims.

Recall from Proposition 2 that, when the settlement cost  $\tau$  is zero, bank  $h$  issues more claims than bank  $\ell$  in a market equilibrium. Part (iv) of Proposition 3 shows that the settlement cost,  $\tau$ , acts in the opposite direction, and induces bank  $h$  to reduce the quantity of claims it issues. In a payments equilibrium, the overall effect from these two countervailing forces is such that, if  $\alpha_h$  is close to  $\bar{\alpha}$ , in equilibrium bank  $h$  issues fewer claims than bank  $\ell$ . Conversely, if  $\alpha_h$  is sufficiently greater than  $\bar{\alpha}$ , bank  $h$  issues more claims than bank  $\ell$ .

Finally, in part (v) of Proposition 3, we show that in some cases when  $\tau > 0$ , bank  $h$  is a lender in the interbank market at time 1. If  $\alpha_h > \hat{\alpha}$ , this implies that bank  $h$  lends more than bank  $\ell$  to the real sector, and is also a lender to the financial sector. As in the case of  $\tau = 0$ , with a general production function and  $\alpha_h \leq \max\{\frac{1}{2}, \hat{\alpha}\}$ , bank  $h$  may be a lender or a borrower in the interbank market. For example, if  $\alpha_\ell = 0$  and the production function is the power function  $f(k) = Ak^y$  with  $A > 0$  and  $y \in (0, 1)$ , at high values of  $A$  (or conversely low values of  $I$ ), bank  $h$  is the borrower in the interbank market when  $\alpha_h$  is close to zero, and the lender when  $\alpha_h$  is close to  $\frac{1}{2}$ . Once again, this highlights the differential effects of the two motives for interbank lending and borrowing: Obtaining investable funds and holding interim liquidity.

Notice that we have assumed that banks are strategic when it comes to lending, but are price-takers in the interbank loan market. The literature on relationship lending (see, e.g., Petersen and Rajan, 1994, or more recently Puri, Rocholl, and Steffen, 2017) finds that informational frictions confer a degree of market power on banks in the loan market. For simplicity, in our model we assume banks are monopolists in their own zones. In the interbank market, while transactions are bilateral, it is reasonable to think that banks have limited market power. For example, Afonso, Kovner, and Schoar (2014) document that the large concentrated lenders in the U.S. interbank market insure borrowers against supply shocks, rather than raising prices when liquidity is low. In our model, we assume banks are price-takers in this market. Overall, we are assuming that it is easier for banks to switch their transaction partners in the interbank loan market than for entrepreneurs to switch lenders.

We have explored some other variants of our model. If entrepreneurs have all the bargaining power in their transactions with banks (so banks are effectively price-takers in the loan market as well), the equilibrium is unchanged. In two versions of the model in which banks are strategic in the interbank market as well, one based on Marschak and Selten (1974) and the other on Gabscewicz and Vial (1972), the liquidity externality we highlight continues to obtain. Finally, if banks  $h$  and  $\ell$  in a given region transact only with each other in the interbank market and can write an enforceable ex ante contract, the second-best outcome is recovered in equilibrium.

### 3.4 Comparative Statics in $\tau$ and $\alpha_h$

As mentioned in Section 2.1, the settlement cost in our model,  $\tau$ , reflects in part the opportunity cost of setting aside collateral for use in payment systems. Costs associated with liquidity management are also an ongoing concern for banks. Efforts to reduce these costs using new tools are being made both by central banks exploring the use of distributed ledger technology<sup>10</sup> by consulting firms that

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<sup>10</sup>Central bank efforts include the Ubin Project in Singapore (<https://www.mas.gov.sg/schemes-and-initiatives/Project-Ubin>) and project Madre from the Banque de France.

offer solutions based on big data.<sup>11</sup>

In our model, reducing the costs of liquidity management corresponds to a reduction in  $\tau$ . We show that such a reduction has opposite effects on the second-best problem and on market equilibrium.

**Proposition 4** *Suppose that  $\tau < \bar{\tau}$  and  $\alpha_h > \bar{\alpha}$ . Consider a small decrease in  $\tau$ , the settlement cost.*

- (i) *In the solution to the second-best problem, there is a strict decrease in  $b_h^s$ , the claims issued by bank  $h$ , and a strict increase in  $b_\ell^s$ , the claims issued by bank  $\ell$ .*
- (ii) *In the market equilibrium, there is a strict increase in the interbank interest rate,  $r^*$ , a strict increase in  $b_h^*$ , the quantity of claims issued by the high-outsourcing bank, and a strict decrease in  $b_\ell^*$ , the quantity of claims issued by the low-outsourcing bank.*

That is, a reduction in the settlement cost  $\tau$  reduces the gap  $b_h^s - b_\ell^s$ , bringing the second-best outcome closer to the first-best outcome. However, it increases the difference in the market equilibrium outcomes,  $b_h^* - b_\ell^*$ . Note that, as shown in Proposition 3 part (iii), if  $\alpha_h < \hat{\alpha}$ , we have  $b_h^* < b_\ell^*$ , so that the equilibrium outcome is closer to the second-best outcome when  $\tau$  reduces. However, if  $\alpha_h$  is sufficiently high (in particular, greater than  $\hat{\alpha}$ ), the converse happens, and a reduction in the settlement cost increases the gap in production between the two zones, and thus exacerbates the externality in the model. This point is illustrated in Example 1 below.

Next, consider the effects of an increase in  $\alpha_h$ , the outsourcing propensity of bank  $h$ , holding all else equal, including  $\alpha_\ell$ , the outsourcing propensity of bank  $\ell$ . In this case, in both the second-best and equilibrium outcomes, the quantity of claims issued by bank  $h$  increases, and the quantity issued by bank  $\ell$  decreases.

**Proposition 5** *Suppose that  $\tau < \bar{\tau}$  and  $\alpha_h > \bar{\alpha}$ . Consider a small increase in  $\alpha_h$ , the outsourcing propensity of bank  $h$ .*

- (i) *In the solution to the second-best problem, there is a strict increase in  $b_h^s$ , the claims issued by bank  $h$ , and a strict decrease in  $b_\ell^s$ , the claims issued by bank  $\ell$ .*
- (ii) *In the market equilibrium, there is a strict increase in the interbank interest rate,  $r^*$ , a strict increase in  $b_h^*$ , the quantity of claims issued by the high-outsourcing bank, and a strict decrease in  $b_\ell^*$ , the quantity of claims issued by the low-outsourcing bank.*

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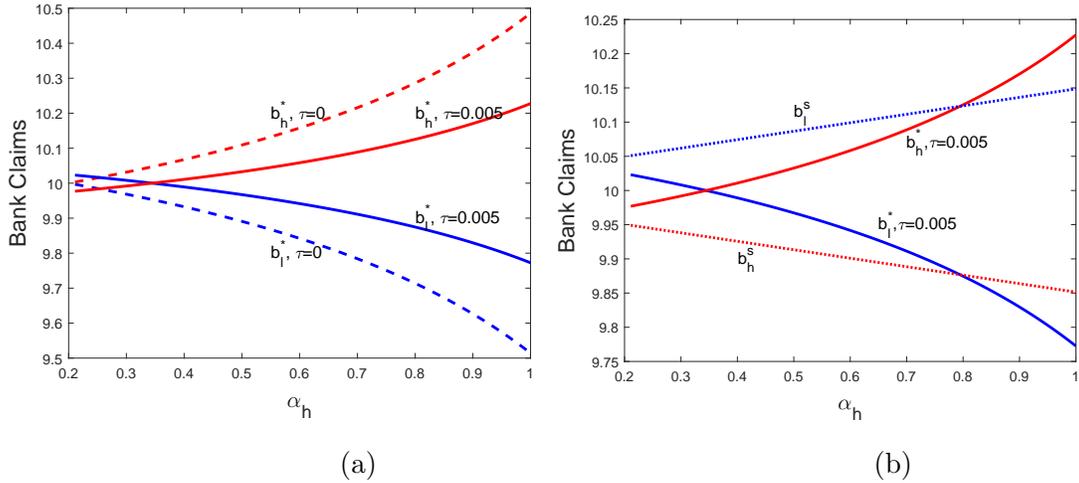
<sup>11</sup>See, for example, [https://www.accenture.com/\\_acnmedia/PDF-98/Accenture-Intraday-Liquidity-Management.pdf](https://www.accenture.com/_acnmedia/PDF-98/Accenture-Intraday-Liquidity-Management.pdf) or the Oliver Wyman report mentioned in footnote 7.

Overall, the externality between zones is heightened if the gap between  $\alpha_h$  and  $\alpha_\ell$  increases, and, when  $\alpha_h$  is high, if the settlement cost  $\tau$  decreases. Conversely, if  $\alpha_h$  is sufficiently close to  $\alpha_\ell$  and  $\tau$  decreases, the externality is reduced.

To illustrate Propositions 4 and 5, and to consider the difference between equilibrium and second-best outcomes when  $\tau$  is positive, we introduce a numerical example.

### Example 1

Set  $f(x) = 2x^{0.8}$ ,  $\lambda = 0.2$ ,  $C = 2.5$ , and  $\alpha_\ell = 0.2$ . We consider the bank claims issued in a payments equilibrium when  $\tau = 0$  and when  $\tau = 0.005$ . With these parameter values,  $\bar{\alpha} = 0.201$ , and we let  $\alpha_h$  vary from  $\bar{\alpha}$  to 1. Figure 2 (a) shows the claims issued by both banks in a market equilibrium for each value of  $\tau$  as  $\alpha_h$  varies. Figure 2 (b) compares the claims issued by each bank in a market equilibrium with the claims issued in the second-best outcome when  $\tau = 0.005$ , again as  $\alpha_h$  varies.



In both figures, we set  $f(x) = 2x^{0.8}$ ,  $\lambda = 0.2$ ,  $C = 2.5$ , and  $\alpha_\ell = 0.2$ . Figure (a) shows the claims issued by bank  $h$  and bank  $\ell$  in a market equilibrium with  $\tau = 0$  (dashed lines) and  $\tau = 50$  basis points (solid lines) as  $\alpha_h$  varies. Figure (b) shows the claims issued by banks  $h$  and  $\ell$  in a market equilibrium (solid lines) and in the second-best outcome (dotted lines) when  $\tau = 0.005$ .

Figure 2: Bank Claims Issued in Market Equilibrium and Second-best Outcome

Two points are worth noting from Figure 2 (a). First, for both values of  $\tau$ , the claims issued by bank  $h$  increase in  $\alpha_h$ . Keeping  $\alpha_\ell$  fixed, an increase in  $\alpha_h$  represents an increase in the liquidity externality that bank  $h$  imposes on bank  $\ell$ . This liquidity externality has real effects, as bank  $h$  invests more when  $\alpha_h$  increases, so that bank  $\ell$  cuts down its own investment. Second, as  $\tau$  increases from 0 to 50 basis points, for each value of  $\alpha_h$  bank  $h$  reduces the claims it issues in equilibrium.

That is, the increase in the settlement cost dampens the incentive of bank  $h$  to issue claims. In the figure, when  $\tau$  is set to 50 basis points, bank  $h$  issues fewer claims than bank  $\ell$  when  $\alpha$  is less than about 0.35, and more claims than bank  $\ell$  when  $\alpha$  exceeds this level. That is, the threshold  $\hat{\alpha}$  identified in Proposition 3 part (iv) is approximately 0.35.

In our model, a change in  $\tau$  affects the planner's second-best solution as well as the market equilibrium. In Figure 2 (b), we plot the bank claims issued in the second-best solution and in the market equilibrium as  $\alpha_h$  varies and  $\tau$  is set to 50 basis points. From the figure, it is clear that in the second-best outcome, bank  $\ell$  issues more claims than bank  $h$  (as stated in Proposition 1). However, in the market equilibrium, we instead have bank  $h$  issuing more claims than bank  $\ell$  when  $\alpha$  exceeds approximately 0.35. ■

A reduction in the settlement cost  $\tau$  certainly mitigates the settlement friction in our model. However, in equilibrium it may exacerbate the liquidity externality. Thus, although the second-best outcome becomes closer to the first-best outcome as  $\tau$  decreases, the equilibrium outcome in fact may diverge further away from the first-best outcome.

All else equal, equilibrium welfare improves as a result of the lower settlement cost, and worsens as a result of the increase in the productivity gap. In various numerical examples, we find that overall equilibrium welfare improves when the settlement cost falls. However, the improvement in total welfare may be accompanied by increased inequality across the zones.

### 3.5 Optimal Division of Settlement Cost Across Payer and Recipient

In our model, we have assumed that the settlement cost at time 2 is borne by the net payer. In this section, we show that to the extent the choice of who pays this cost is determined by policy, a planner would indeed choose to impose this cost on the net payer.

To extend our base model, suppose now that the settlement cost can be shared between the net payer and the net recipient at date 2. Specifically, the net payer pays a fraction  $\gamma$  of the total settlement cost and the net recipient pays a fraction  $1 - \gamma$ , where  $\gamma \in [0, 1]$ . The planner chooses  $\gamma$  to maximize the second-best objective function  $f(b_h^*) + f(b_\ell^*) - b_h^* - b_\ell^* - \tau \max\{\alpha_h b_h^* - \alpha_\ell b_\ell^*, 0\}$ . After the planner chooses  $\gamma$ , the variables  $r^*$ ,  $b_h^*$ , and  $b_\ell^*$  are determined in equilibrium.

We have shown in Proposition 2 that, when  $\tau = 0$ , bank  $h$  makes a net transfer to bank  $\ell$  at date 2. For values of  $\tau$  close to zero, the equilibrium too will feature bank  $h$  being a net payer and bank  $\ell$  being a net recipient at date 2.

Recall from Proposition 3 that when  $\alpha_h > \bar{\alpha}$ , when  $\gamma = 1$  (i.e., the payer at date 2 bears the entire settlement cost), in equilibrium bank  $h$  makes a transfer to bank  $\ell$ . In the proof of Proposition 6, we show that when  $\alpha > \bar{\alpha}$  this property of equilibrium is maintained for all  $\gamma \in [0, 1]$ . Under

these circumstances, we can write the profit functions of banks  $h$  and  $\ell$  respectively as:

$$\pi_h = f(b_h) - b_h - rz_h - \gamma(\alpha_h b_h - \alpha_\ell b_\ell) \quad (13)$$

$$\pi_\ell = f(b_\ell) - b_\ell - rz_\ell - (1 - \gamma)(\alpha_h b_h - \alpha_\ell b_\ell). \quad (14)$$

The banks' best responses may now be determined from the respective first-order conditions. Rewriting  $b_h^-$  and  $b_\ell^+$  as functions of  $\gamma$ , we now have

$$b_h^-(\gamma) = g(1 + r\lambda(1 - \alpha_h) + \gamma\tau\alpha_h) \quad (15)$$

$$b_\ell^+(\gamma) = g(1 + r\lambda(1 - \alpha_\ell) - (1 - \gamma)\tau\alpha_\ell), \quad (16)$$

where  $g(x) = f'^{-1}(x)$  as before. The equilibrium interest rate  $r^*$  will also depend on the chosen value of  $\gamma$ .

We show that it is optimal for the planner to set  $\gamma^* = 1$ ; i.e., to require the net payer to pay the entire settlement cost. In the proof of Proposition 6, we show that when  $\alpha > \bar{\alpha}$ , in equilibrium  $b_h^*$  is decreasing in  $\gamma$  and  $b_\ell^*$  is increasing in  $\gamma$ . In Proposition 3 part (ii), we have shown that when  $\gamma = 1$ , we already have  $b_h^* > b_h^s$  (that is, the equilibrium quantity of bank  $h$  claims exceeds the quantity in the planner's second-best solution), and  $b_\ell^* < b_\ell^s$ . Any reduction in  $\gamma$  merely increases the size of the distortion relative to the second-best outcome.

**Proposition 6** *If  $\alpha > \bar{\alpha}$ , the planner sets  $\gamma^* = 1$ . That is, the planner optimally imposes the entire settlement cost on the net payer at date 2.*

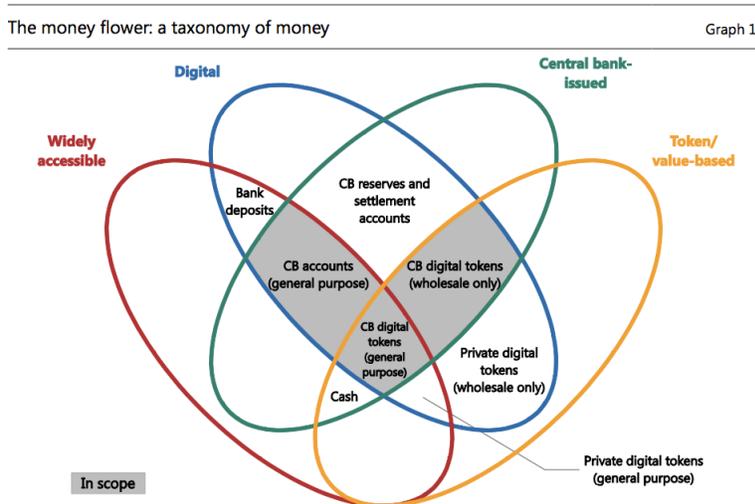
The intuition for this result is as follows. Recall that, in the second-best outcome exhibited in Proposition 1, the difference in marginal products between banks  $h$  and  $\ell$  is  $\tau(\alpha_h + \alpha_\ell)$ . If the claims of bank  $h$  are increased by \$1, bank  $h$  has to make an additional transfer of  $\alpha_h$  to bank  $\ell$  at time 2. However, by market-clearing, bank  $\ell$  reduces its claims by \$1, which reduces the amount it owes to bank  $\ell$  by  $\alpha_\ell$ . Thus, the net additional transfer of bank  $h$  (or in other words the marginal settlement cost to the planner of increasing the claims of bank  $h$  by \$1) amounts to  $\tau(\alpha_h + \alpha_\ell)$ .

Ideally, the planner would like banks to internalize this marginal settlement cost into their own-decision making. However, the best response conditions of the bank imply that bank  $h$  takes into account the cost  $\gamma\tau\alpha_h$ , and bank  $\ell$  takes into account  $(1 - \gamma)\tau\alpha_\ell$ . As  $\alpha_h > \alpha_\ell$ , setting  $\gamma^* = 1$  maximizes the amount of the marginal settlement cost that is internalized by the two banks.

## 4 Application: Central Bank Digital Currency

The phrase ‘‘central bank digital currency’’ (CBDC) is used to refer to different forms of digital fiat money. In a Bank for International Settlements (BIS) survey, Barontini and Holden (2019)

mention that 70% of the world’s central banks are exploring or will soon be exploring CBDC. Bech and Garratt (2017) provide a comprehensive taxonomy that has been adapted and has come to be known as the “BIS money flower” (see Figure 3).



This figure is taken from Barontini and Holden (2019), and was adapted from Bech and Garratt (2017).

Figure 3: **The BIS money flower**

We use the phrase CBDC to refer to token-based digital currencies, or cryptocurrencies, issued by a central bank. Currently, CBDC projects are still at the planning stage, and there has not been large-scale adoption. Many possible variants are being discussed.<sup>12</sup> We therefore focus on the broad features of two kinds of proposed CBDC: wholesale and retail CBDC.

Only designated financial institutions will have access to wholesale CBDC. Most projects on wholesale CBDC have been initiated by central banks, but some are from the private sector. Among the former are Project Jasper (Bank of Canada), Project Ubin (Monetary Authority of Singapore), and the Stella Project (joint Bank of Japan and the ECB). Among the latter are “Utility Settlement Coin” by a private consortium initiated by UBS with Clearmatics. The distinction with the current form of central bank reserves is that transactions in wholesale CBDC will be recorded on a distributed ledger. The projected benefit of wholesale CBDC is that interbank transactions will require less upfront collateral, and hence may be more efficient for banks.

In the context of our model, we interpret wholesale CBDC as an innovation that reduces the settlement cost  $\tau$ . Proposition 4 applies to this case—if the degree to which projects are sourced

<sup>12</sup>For details, see “Central Bank Digital Currencies,” Bank for International Settlements Committee on Payments and Market Infrastructures, March 2018, available at <https://www.bis.org/cpmi/publ/d174.pdf>.

across regions is widely heterogeneous, regional inequalities in production may become worse when the settlement cost is reduced.

Retail CBDC is envisioned as a new form of central bank money. Consumers will directly hold accounts at the central bank, and can transact with each other using retail CBDC. Examples of retail CBDC include both pilot projects (such as the e-krona in Sweden and the e-peso pilot in Uruguay) and partial adoption such as the e-dinar in Tunisia and the SOV issued by the Marshall Islands. In our model, the availability of retail CBDC reduces the need for cash in the economy. Thus, households deposit less cash in banks, and also withdraw less cash to meet their interim liquidity needs.

As of now, it remains an open question whether retail CBDC will drive out current forms of money completely or will co-exist with cash and bank deposits. Agur, Ari, and Dell’Ariccia (2019) consider a model in which some consumers use retail CBDC, whereas others continue to use existing monetary instruments. In our model, the implication would be that some consumers stop using cash. Suppose retail CBDC is issued, and a proportion  $\phi$  of consumers switch over to it. Bank deposits fall to  $(1 - \phi)C$ , and the proportion of consumers needing interim liquidity at date  $t = 1$  falls to  $(1 - \phi)\lambda$ . Total investable funds may now be defined as  $\tilde{I} = \frac{(1-\phi)(1-\lambda)C}{(1-\phi)\lambda} = I$ . This quantity is invariant to  $\phi$ , so the partial use of retail CBDC has no effect on lending in our model.

## 5 Conclusion

Ensuring a stable and efficient payment system is one of the core principles of banking supervision reforms. For example, the Basel Committee on Banking Supervision (2008) has encouraged the adoption of comprehensive rules on liquidity risk in payment systems. In this paper we highlight that a bank’s liquidity needs can depend in part on the actions of other banks in the payment system. Even though banks can freely trade reserves in an interbank market, strategic considerations affect where credit is allocated, and thus the productive efficiency of the economy. In other words, the amount a bank lends is critically affected by the fact that it functions as a part of the payment system.

A standard intuition is that in the presence of an interbank market to reallocate resources, the marginal product of capital and hence investment will equalize across different production regions. As we have demonstrated, if banks are also responsible for ex post settlement in the payment system, this will not occur. Indeed, while we have illustrated cross-sectional differences in investment, with a concave production technology aggregate output is also lower than an ideal benchmark level.

In our model, the payment system creates a liquidity externality that distorts lending away from first-best levels. The settlement cost  $\tau$  dampens the extent of this externality, as the cost is

incurred by the bank that is a net payer at the end of the game. Reducing the settlement cost (perhaps through wholesale CBDC) is on the whole welfare-improving, but does have distributional implications, and may exacerbate inequalities in lending across zones.

## Appendix: Proofs

### Proof of Lemma 1

The liquidity constraints on the planner's problem are:  $z_h \geq \lambda((1 - \alpha_h)b_h + \alpha_\ell b_\ell - I)$  and  $z_\ell \geq \lambda((1 - \alpha_\ell)b_\ell + \alpha_h b_h - I)$ . Summing the two, we obtain  $z_h + z_\ell \geq \lambda(b_h + b_\ell - 2I)$ . In conjunction with the market-clearing condition  $z_h + z_\ell = 0$ , this implies that  $b_h + b_\ell \leq 2I$ .

Consider the following relaxed problem for the planner:

$$\begin{aligned} \max_{b_h, b_\ell} & f(b_h) + f(b_\ell) - b_h - b_\ell \\ \text{subject to:} & \quad b_h + b_\ell \leq 2I. \end{aligned}$$

At the optimum, the constraint must bind. Suppose not; then, at least one of  $b_h$  or  $b_\ell$  must be less than  $\beta_h = \frac{\alpha_h}{\alpha_h + \alpha_\ell} 2I < 2I$ . Suppose  $b_\ell < \beta_h$ . From Assumption 1 (i), it follows that a small increase in  $b_\ell$  strictly increases the objective function. A similar argument holds if  $b_h < \beta_h$ . Thus, the constraint must bind.

As  $f(\cdot)$  is concave, and the first-best problem is completely symmetric in banks  $h$  and  $\ell$ , it is now immediate that the solution involves  $b_\ell^f = b_h^f$ . Now, the aggregate resource constraint is  $z_\ell + z_h = 0$ , from which it follows that it must be that  $b_\ell^f = b_h^f = I$ . ■

### Proof of Proposition 1

Consider two matched banks in the same region. The total surplus the planner generates from these two banks is

$$\Pi = f(b_\ell) - b_\ell + f(b_h) - b_h - \tau|\alpha_\ell b_\ell - \alpha_h b_h|. \quad (17)$$

The constraints are the market-clearing condition  $z_\ell + z_h = 0$ , and the liquidity constraints  $z_h \geq \lambda((1 - \alpha_h)b_h + \alpha_\ell b_\ell - I)$  and  $z_\ell \geq \lambda((1 - \alpha_\ell)b_\ell + \alpha_h b_h - I)$ .

As in the proof of Lemma 1, consider the relaxed problem in which the liquidity constraints are replaced by the constraint  $b_h + b_\ell \leq 2I$ . We first show that this constraint must bind at the optimum. Suppose not, so that  $b_h + b_\ell < 2I$ . There are three cases to consider:

(i)  $\alpha_\ell b_\ell > \alpha_h b_h$ . In this case, as  $\alpha_\ell < \alpha_h$ , it must be that  $b_h < \beta_h$ . Now, by Assumption 1 (i), a small increase in  $b_h$  must increase the value of the objective function, as  $f(b_h) - b_h$  increases and the transaction cost term decreases.

(ii)  $\alpha_\ell b_\ell < \alpha_h b_h$ . Observe that  $\beta_h + \beta_\ell = 2I$  and that  $\alpha_\ell \beta_h = \alpha_h b_\ell$ . Thus,  $\alpha_\ell b_\ell < \alpha_h b_h$  implies that  $b_\ell < \beta_h$ . Now, by Assumption 1 (i), a small increase in  $b_\ell$  must increase the value of the objective function, as  $f(b_\ell) - b_\ell$  increases and the transaction cost term decreases.

(iii)  $\alpha_\ell b_\ell = \alpha_h b_h$ . In this case, it must be that  $b_\ell$  and  $b_h$  are each strictly less than  $\beta_h$ . Thus,

increasing  $b_\ell$  by a small amount  $\epsilon > 0$  and  $b_h$  by an amount  $\frac{\alpha_\ell}{\alpha_h}\epsilon$  strictly increases output, and keeps the transaction cost at zero. Thus, the objective function increases.

Therefore, at the optimum, it must be that  $b_h + b_\ell = 2I$ . Now, suppose that in the planner's solution, at date 2 bank  $\ell$  needs to make a payment transfer to bank  $h$ ; that is, suppose  $\alpha_\ell b_\ell > \alpha_h b_h$ . As  $\alpha_\ell < \alpha_h$ , it must be that  $b_\ell > b_h$ . Consider the following adjustment: Reduce  $b_\ell$  by a small amount  $\epsilon > 0$  and increase  $b_h$  by  $\epsilon$ . Then, total output increases at the rate  $f'(b_h) - f'(b_\ell) > 0$ . Further, the aggregate payment cost falls by  $\tau(\alpha_\ell + \alpha_h)$ . Therefore, a strict improvement in the objective function is obtained, contradicting the assumption that we were at an optimum.

Hence, in the planner's solution, it must be that  $\alpha_h b_h \geq \alpha_\ell b_\ell$ . Consider some  $b_\ell$  and  $b_h$  that satisfy both the last inequality and the aggregate resource constraint  $b_\ell + b_h = 2I$ . Reduce  $b_\ell$  by some small amount  $\epsilon > 0$  and increase  $b_h$  by  $\epsilon$ . The rate of change of the objective function is  $f'(b_h) - f'(b_\ell) - \tau(\alpha_h + \alpha_\ell)$ . There are two possibilities:

- (a) The rate of change in the objective function is strictly positive. In this case, it must be that in equilibrium  $\alpha_h b_h > \alpha_\ell b_\ell$ , and the optimal solution is obtained when the rate of change of the objective function exactly hits zero; that is, at the point at which  $f'(b_h^s) - f'(b_\ell^s) = \tau(\alpha_h + \alpha_\ell)$ .
- (b) The rate of change in the objective function is weakly negative. In this case, the optimal solution entails  $\alpha_h b_h^s = \alpha_\ell b_\ell^s$ . Using the market-clearing constraint, we obtain that  $b_\ell^s = \beta_h$  and  $b_h^s = \beta_\ell$ . Hence, for this case to occur, it must be that  $f'(\beta_\ell) - f'(\beta_h) \leq \tau(\alpha_h + \alpha_\ell)$ , or  $\tau \geq \frac{f'(\beta_\ell) - f'(\beta_h)}{\alpha_h + \alpha_\ell} = \bar{\tau}$ .

Therefore, if  $\tau < \bar{\tau}$ , we are in case (a). In this case, we have  $f'(b_h^s) - f'(b_\ell^s) = \tau(\alpha_h + \alpha_\ell)$  and  $b_\ell^s + b_h^s = 2I$ . The comparative statics in part (a) (i) of the proposition now follow. Further, as  $\alpha_h b_h^s > \alpha_\ell b_\ell^s$ , bank  $h$  makes a net transfer to bank  $\ell$  at time 2.

Conversely, if  $\tau \geq \bar{\tau}$ , we are in case (b), so that  $b_\ell^s = \beta_h$  and  $b_h^s = \beta_\ell$ . Observe that  $\alpha_h \beta_\ell = \alpha_\ell \beta_h$ . Therefore, in this case, there are no net transfers between banks at time 2. ■

## Proof of Lemma 2

Suppose  $r > 0$ . Then, it follows that the interim liquidity constraint in equation (2) must bind. That is, it must be the case that  $z_i = \lambda((1 - \alpha)b_i + b_{-i} - I)$ . If not, the profit of the bank can be trivially increased by reducing  $z_i$  by a small amount.

Now, consider two matched banks  $i$  and  $-i$ . Suppose first that  $\alpha_i b_i > \alpha_{-i} b_{-i}$ ; that is,  $b_i > \frac{\alpha_{-i}}{\alpha_i} b_{-i}$ . Then, bank  $i$  is a net payer in the interbank payment system at date 2. Hence, its profit function is

$$\pi_i = f(b_i) - b_i - r\lambda\{(1 - \alpha_i)b_i + \alpha_{-i}b_{-i} - I\} - \tau(\alpha_i b_i - \alpha_{-i} b_{-i}), \quad (18)$$

where we have substituted in  $z_i = \lambda((1 - \alpha)b_i + b_{-i} - I)$ . The first-order condition in  $b_i$  yields

$$f'(b_i) - 1 - \lambda r(1 - \alpha_i) - \tau \alpha_i = 0 \quad (19)$$

$$b_i = g\left(1 + \lambda r(1 - \alpha_i) + \tau \alpha_i\right) \equiv b_i^-. \quad (20)$$

Concavity of  $f(\cdot)$  ensures that the second-order condition is satisfied. Thus, if  $b_i^- > \frac{\alpha_{-i}}{\alpha_i} b_{-i}$ , then  $b_i^* = b_i^-$ .

Next, suppose that  $\alpha_i b_i < \alpha_{-i} b_{-i}$ ; that is,  $b_i > \frac{\alpha_{-i}}{\alpha_i} b_{-i}$ . Then, bank  $i$  is a net recipient at date 2. Hence, its profit function is

$$\pi_i = f(b_i) - b_i - r\lambda\{(1 - \alpha_i)b_i + \alpha_{-i}b_{-i} - I\}, \quad (21)$$

again after substituting in  $z_i = \lambda((1 - \alpha)b_i + b_{-i} - I)$ . The first-order condition in  $b_i$  yields

$$f'(b_i) - 1 - \lambda r(1 - \alpha_i) = 0 \quad (22)$$

$$b_i = g\left(1 + \lambda r(1 - \alpha_i)\right) \equiv b_i^+. \quad (23)$$

Concavity of  $f(\cdot)$  ensures that the second-order condition is satisfied. Thus, if  $b_i^+ < \frac{\alpha_{-i}}{\alpha_i} b_{-i}$ , then  $b_i^* = b_i^+$ .

Finally, if  $\frac{\alpha_{-i}}{\alpha_i} b_{-i} \in [b_i^-, b_i^+]$ , then it follows that  $b_i^* = \frac{\alpha_{-i}}{\alpha_i} b_{-i}$ .

Given that the interim liquidity constraint binds, we immediately have  $z_i^* = \lambda((1 - \alpha)b_i^* + b_{-i} - I)$ . ■

## Proof of Proposition 2

(i) When  $\tau = 0$ , we have  $b_i^- = b_i^+ = g(1 + r\lambda(1 - \alpha_i))$  for each bank  $i = \ell, h$ . That is, given  $r$ , the best response of bank  $i$  is to set  $b_i = g(1 + r\lambda(1 - \alpha_i))$ . Observe that  $b_i$  is strictly decreasing in  $r$ , and so the excess demand function  $b_\ell + b_h - 2I$  is strictly decreasing in  $r$ .

Now, when  $r = 0$  and  $\tau = 0$ , the profit function of bank  $i$  is  $\pi_i = f(b_i) - b_i$ , which is independent of both  $z_i$  and  $b_{-i}$ . Thus, with each bank optimizing, we have  $b_\ell = b_h = g(1)$ . As  $f'(I) > 1$  by Assumption 1 part (i), at  $r = 0$  there is an excess demand for funds. Observe that  $b_i^-$  and  $b_i^+$  are each strictly decreasing in  $r$ . Hence, it must be that  $r > 0$ . As noted in Lemma 2, this further implies that the interim liquidity constraint for each bank (2) is binding, so that  $z_i = \lambda((1 - \alpha)b_i + b_{-i} - I)$  for each  $i$ . The market-clearing constraint now implies that  $b_h + b_\ell = 2I$ .

Now, at  $r = \frac{f'(I) - 1}{\lambda(1 - \alpha_h)}$ , we have  $b_h = I$ . However,  $b_\ell = g(1 + r\lambda(1 - \alpha_\ell)) < g(1 + r\lambda(1 - \alpha_h)) = b_h$ , so at this interest rate there is excess supply of funds. Hence, there exists an interest rate  $r^* \in (0, \frac{f'(I) - 1}{\lambda(1 - \alpha_h)})$  such that  $b_h^* + b_\ell^* = 2I$ . Further, as  $b_h^*$  and  $b_\ell^*$  are strictly decreasing in  $r$ , the interest rate  $r^*$  is unique.

(ii) As  $r^* > 0$ , it follows that  $b_h^* = g(1 + r^*\lambda(1 - \alpha_h)) > b_\ell^* = g(1 + r^*\lambda(1 - \alpha_\ell))$  when  $\alpha_h > \alpha_\ell$ . The market-clearing constraint  $b_h^* + b_\ell^* = 2I$  now implies that  $b_h^* > I > b_\ell^*$ .

(iii) As  $b_h^* > b_\ell^*$  and  $\alpha_h > \alpha_\ell$ , it follows immediately that  $\alpha_h b_h^* > \alpha_\ell b_\ell^*$ ; that is, bank  $h$  makes a net transfer to bank  $\ell$  at time time 2.

(iv) We have  $z_h^* = \lambda((1 - \alpha_h)b_h^* + \alpha_\ell b_\ell^* - I)$  and  $z_\ell^* = \lambda((1 - \alpha_\ell)b_\ell^* + \alpha_h b_h^* - I)$ .

Therefore,  $z_\ell^* > z_h^* \iff (1 - \alpha_\ell)b_\ell^* + \alpha_h b_h^* > (1 - \alpha_h)b_h^* + \alpha_\ell b_\ell^*$ , or  $(1 - 2\alpha_\ell)b_\ell^* > (1 - 2\alpha_h)b_h^*$ .

Consider the following two cases:

(a)  $f(x) = A \ln x$  where  $A > 0$ . In this case, when  $\tau = 0$  we have  $b_h^* = \frac{A}{1+r^*\lambda(1-\alpha_h)}$  and  $b_\ell^* = \frac{A}{1+r^*\lambda(1-\alpha_\ell)}$ . Therefore, the condition  $(1 - 2\alpha_\ell)b_\ell^* > (1 - 2\alpha_h)b_h^*$  holds if and only if  $\frac{1-2\alpha_\ell}{1+r^*\lambda(1-\alpha_\ell)} > \frac{1-2\alpha_h}{1+r^*\lambda(1-\alpha_h)}$ , or  $(\alpha_h - \alpha_\ell)(2 + r^*\lambda) > 0$ . Now,  $r^* > 0$  given Assumption 1 part (i), and  $\alpha_h > \alpha_\ell$ . Thus, the last inequality holds, so that  $z_\ell^* > z_h^*$ . Market-clearing now implies that  $z_\ell^* > 0 > z_h^*$ .

(b)  $\alpha_h \geq \frac{1}{2}$ . If  $\alpha_\ell \leq \frac{1}{2}$ , it is immediate that  $(1 - 2\alpha_\ell)b_\ell^* > (1 - 2\alpha_h)b_h^*$ . If  $\alpha_\ell > \frac{1}{2}$ , then  $\alpha_h > \alpha_\ell \implies 0 > 1 - 2\alpha_\ell > 1 - 2\alpha_h$ . Now,  $b_\ell^* < b_h^*$  implies that  $(1 - 2\alpha_\ell)b_\ell^* > (1 - 2\alpha_h)b_h^*$ . Hence, for  $\alpha_h \geq \frac{1}{2}$ , we have  $z_\ell^* > z_h^*$ . Market-clearing now implies that  $z_\ell^* > 0 > z_h^*$ .

■

### Proof of Lemma 3

*Step 1:* We first define  $\underline{r}$  and  $\bar{r}$ , and show the existence of a  $\bar{\alpha}$  at which  $\underline{r}(\bar{\alpha}) = \bar{r}(\bar{\alpha})$ .

In what follows, we keep  $\alpha_\ell$  fixed and vary  $\alpha_h$ . For notational convenience, in this step alone, let  $\alpha$  denote  $\alpha_h$ . Then,  $\beta_\ell = \frac{2\alpha_\ell}{\alpha_\ell + \alpha}I$  and  $\beta_h = \frac{2\alpha}{\alpha_\ell + \alpha}I$ .

Define

$$\underline{r}(\alpha \mid \alpha_\ell, \tau, I) = \frac{f'(\beta_\ell) - 1 - \tau\alpha}{\lambda(1 - \alpha)} \quad (24)$$

$$\bar{r}(\alpha \mid \alpha_\ell, \tau, I) = \frac{f'(\beta_h) - 1}{\lambda(1 - \alpha_\ell)} \quad (25)$$

Then,  $\underline{r}$  is the interest rate at which  $b_h^- = \beta_\ell$ . If  $r > \underline{r}$ , we have  $b_h^- < \beta_\ell$  and if  $r < \underline{r}$  we have  $b_h^- > \beta_\ell$ . Similarly,  $\bar{r}$  is the interest rate at which  $b_\ell^+ = \beta_h$ , with  $b_\ell^+ > \beta_h$  when  $r < \bar{r}$  and  $b_\ell^+ < \beta_h$  when  $r > \bar{r}$ . Observe that  $\underline{r}(\alpha) > 0$  as  $f'(\beta_h) > 1$  (by Assumption 1 (i)), and  $\underline{r}(\alpha) > 0$  as  $f'(\beta_\ell) > f'(\beta_h) > 1 + \tau \geq 1 + \tau\alpha$ , where  $f'(\beta_h) > 1 + \tau$  follows from Assumption 1 (ii).

Suppose that  $\alpha = \alpha_\ell$ ; i.e., the outsourcing propensities are the same across the two banks. Observe that  $\underline{r}(\alpha_\ell \mid \cdot) \leq \bar{r}(\alpha_\ell \mid \cdot)$ , with strict inequality when  $\tau > 0$  and  $\alpha_\ell > 0$ .

Now, consider  $\alpha$  increasing, starting at  $\alpha = \alpha_\ell$ . As  $\frac{2\alpha}{\alpha_\ell + \alpha}$  is increasing in  $\alpha$  and  $g(\cdot)$  is a decreasing function, it follows that  $\bar{r}(\alpha | \cdot)$  is strictly decreasing in  $\alpha$ . The partial derivative of  $\underline{r}(\alpha)$  with respect to  $\alpha$  is

$$\frac{\partial \underline{r}(\alpha)}{\partial \alpha} = \frac{(1 - \alpha) \left( -f''(\beta_\ell) \frac{\beta_\ell}{\alpha_\ell + \alpha} - \tau \right) + \left( f'(\beta_\ell) - 1 - \tau \alpha \right)}{\lambda(1 - \alpha)^2} \quad (26)$$

Now, under Assumption 1 part (ii), we have  $\tau < -f''(\beta_\ell) \frac{\beta_\ell}{\alpha_\ell + \alpha_h}$  and  $f'(\beta_h) > 1 + \tau$ . The left-hand side of the latter inequality is weakly less than  $f'(\beta_\ell)$ , and the right-hand side is strictly greater than  $1 + \tau\alpha$ . Therefore, both terms contained in the large parentheses in the numerator of equation (26) are strictly positive, so that  $\underline{r}(\alpha_h | \cdot)$  is strictly increasing in  $\alpha_h$ .

Finally, observe that as  $\alpha_h \rightarrow 1$ ,  $\underline{r}(\alpha | \cdot) \rightarrow \infty$ , whereas  $\bar{r}(\alpha | \cdot)$  stays finite.

It now follows that there exists an  $\bar{\alpha} \in (\alpha_\ell, 1)$  such that  $\underline{r}(\alpha | \cdot) \leq \bar{r}(\alpha | \cdot)$  for  $\alpha \leq \bar{\alpha}$ , with  $\underline{r}(\alpha | \cdot) > \bar{r}(\alpha | \cdot)$  for  $\alpha_h > \bar{\alpha}$ .

*Step 2:* We now show that  $\alpha_h \leq \bar{\alpha}$  implies a no-payments equilibrium.

Suppose that  $\alpha_h < \bar{\alpha}$ , so that  $\underline{r}(\alpha_h | \cdot) \leq \bar{r}(\alpha_h | \cdot)$ . Consider any  $r \in [\underline{r}, \bar{r}]$ . We have shown that, for such an  $r$ , we have  $b_h^- \leq \beta_\ell < \beta_h \leq b_\ell^+$ . Observe that (i)  $\alpha_\ell \beta_h = \alpha_h \beta_\ell$  and (ii)  $\beta_\ell + \beta_h = 2I$ .

Set  $b_\ell^* = \beta_h$  and  $b_h^* = \beta_\ell$ . Because  $\beta_h + \beta_\ell = 2I$ , the market-clearing condition is satisfied.

Now, observe that  $\underline{r}(\alpha_h) > 0$ , as part (ii) of Assumption 1 implies that  $f'(\beta_\ell) > 1 + \tau\alpha$ . Hence,  $r > 0$ , which implies that  $b_h^- > b_\ell^-$  and  $b_h^+ > b_\ell^+$ . Therefore, we have  $b_\ell^* = \beta_h \in (b_\ell^-, b_\ell^+]$  and  $b_h^* = \beta_\ell \in [b_h^-, b_h^+)$ . Therefore, from Lemma 2, each bank is playing a best response, and hence we have a Nash equilibrium in the banks' game.

Finally, we show that there is no equilibrium with  $r < \underline{r}(\alpha_h)$  or  $r > \bar{r}(\alpha_h)$ . Suppose there is an equilibrium with  $r < \underline{r}(\alpha_h)$ . As mentioned earlier,  $r < \underline{r}(\alpha_h) \implies b_h^- > \beta_\ell$ . From Lemma 2, we have  $b_h^* \in [b_h^-, b_h^+]$ , so in this case it must be that  $b_h^* > \beta_\ell$ . Now, the market-clearing constraint implies that  $b_\ell^* < \beta_h$ . Hence, as  $\alpha_\ell \beta_h = \alpha_h \beta_\ell$ , it must be that  $\alpha_\ell b_\ell^* < \alpha_h b_h^*$ . But then Lemma 2 implies that  $b_\ell^* = b_\ell^+$ , and when  $r < \underline{r}(\alpha_h) < \bar{r}(\alpha_h)$ , we have  $b_\ell^+ > \beta_h$ , which is a contradiction. Hence, there cannot be an equilibrium with  $r < \underline{r}(\alpha_h)$ .

Next, suppose there is an equilibrium with  $r > \bar{r}(\alpha_h)$ . As mentioned earlier,  $r > \bar{r}(\alpha_h) \implies b_\ell^+ < \beta_h$ . From Lemma 2, we have  $b_\ell^* \in [b_\ell^-, b_\ell^+]$ , so in this case it must be that  $b_\ell^* < \beta_h$ . Now, the market-clearing constraint implies that  $b_h^* > \beta_\ell$ . Hence, as  $\alpha_\ell \beta_h = \alpha_h \beta_\ell$ , it must be that  $\alpha_\ell b_\ell^* < \alpha_h b_h^*$ . But then Lemma 2 implies that  $b_h^* = b_h^-$ , and when  $r > \bar{r}(\alpha_h) > \underline{r}(\alpha_h)$ , we have  $b_h^- < \beta_\ell$ , which is a contradiction. Hence, there cannot be an equilibrium with  $r > \bar{r}(\alpha_h)$ . ■

### Proof of Proposition 3

(i) There are only three possibilities in equilibrium:

- (a)  $\alpha_\ell b_\ell^* = \alpha_h b_h^*$ , so that there are no net settlement transfers at date 2. In this case, as argued above, the market-clearing constraint implies that  $b_\ell^* = \beta_h$  and  $b_h^* = \beta_\ell$ .
- (b)  $\alpha_\ell b_\ell^* > \alpha_h b_h^*$ , so that bank  $\ell$  must make a settlement transfer to bank  $h$  at date 2. In this case, Lemma 2 implies that  $b_\ell^* = b_\ell^-$  and  $b_h^* = b_h^+$ .
- (c)  $\alpha_\ell b_\ell^* < \alpha_h b_h^*$ , so that bank  $h$  must make a settlement transfer to bank  $\ell$  at date 2. In this case, Lemma 2 implies that  $b_\ell^* = b_\ell^+$  and  $b_h^* = b_h^-$ .

Now, suppose that  $\alpha_h > \bar{\alpha}$ . Then, as argued in part (i) above, we have  $\underline{r}(\alpha_h) > \bar{r}(\alpha_h)$ . We first rule out the possibility of an equilibrium with  $r \geq \underline{r}(\alpha_h)$  or  $r \leq \bar{r}(\alpha_h)$ , and then show that an equilibrium exists for some  $r \in (\bar{r}(\alpha_h), \underline{r}(\alpha_h))$ .

Suppose that the equilibrium interest rate satisfies  $r \geq \underline{r}(\alpha_h) > \bar{r}(\alpha_h)$ . Then, given the definition of  $\bar{r}(\alpha_h)$ , we have  $b_\ell^+ < \beta_h$ . However,  $b_\ell^* \in [b_\ell^-, b_\ell^+]$  in all cases by Lemma 2. Hence, there cannot be an equilibrium in which  $b_\ell^* = \beta_h$ , ruling out (a) above as a candidate for equilibrium.

Next, suppose that with  $r \geq \underline{r}(\alpha_h)$  there is an equilibrium satisfying case (b) above. Observe that  $r > \bar{r}(\alpha_h)$  implies that  $r > 0$ . Hence, from the definition of  $b_i^-$  in equation (10),  $b_\ell^- < b_h^-$ , and as  $r \geq \underline{r}(\alpha)$  we have  $b_h^- \leq \beta_\ell$ . Therefore,  $b_\ell^* = b_\ell^- < b_h^- \leq \beta_\ell$ . Market-clearing now implies that we must have  $b_h^* > \beta_h$ . But then we must have  $\alpha_\ell b_\ell^* < \alpha_h b_h^*$ , which directly contradicts the assumption in case (b) that  $\alpha_\ell b_\ell^* > \alpha_h b_h^*$ .

Finally, as  $b_h^- \leq \beta_\ell$  and  $b_\ell^+ < \beta_h$ , the market-clearing constraint is immediately violated in case (c) above, so there cannot be such an equilibrium either. Hence, there cannot be an equilibrium with  $r \geq \underline{r}(\alpha_h)$ .

Next, suppose that the equilibrium interest rate satisfies  $r \leq \bar{r}(\alpha_h) < \underline{r}(\alpha_h)$ . Then, as argued in part (i) above, we have  $b_h^+ > \beta_h$ , and  $\beta_h > \beta_\ell$  by definition. However,  $b_h^* \in [b_h^-, b_h^+]$  in all cases by Lemma 2. Hence, there cannot be an equilibrium in which  $b_h^* = \beta_\ell$ , ruling out (a) above as a candidate for equilibrium.

Next, suppose that with  $r \leq \bar{r}(\alpha_h)$  there is an equilibrium satisfying case (b) above. Observe that when  $r > 0$ ,  $b_i^+$  is increasing in  $\alpha_i$ . Hence, from the definition of  $b_i^+$  in equation (9), we have  $b_h^+ > b_\ell^+$ , and as  $r \leq \bar{r}(\alpha)$  we have  $b_\ell^+ \geq \beta_h$ . Therefore,  $b_h^* = b_h^+ > b_h^- \geq \beta_h$ . Market-clearing now implies that we must have  $b_\ell^* < \beta_\ell$ . But then we must have  $\alpha_\ell b_\ell^* < \alpha_h b_h^*$ , which directly contradicts the assumption in case (b) that  $\alpha_\ell b_\ell^* > \alpha_h b_h^*$ .

Finally, as  $b_\ell^- > \beta_\ell$  and  $b_\ell^+ \geq \beta_h$ , the market-clearing constraint is immediately violated in case (c) above, so there cannot be such an equilibrium either. Hence, there cannot be an equilibrium with  $r \leq \bar{r}(\alpha_h)$ .

Therefore, if an equilibrium exists, it must have an equilibrium interest rate  $r \in (\bar{r}(\alpha_h), \underline{r}(\alpha_h))$ . At any such interest rate, as  $r < \underline{r}(\alpha_h)$ , we have  $b_h^- > \beta_\ell$ . As  $b_h^* \in [b_h^-, b_h^+]$ , this rules out case (a)

as an equilibrium. Further,  $b_h^* \in [b_h^-, b_h^+]$  implies from market-clearing that  $b_\ell^* < \beta_\ell$ , which violates the condition in case (b) that  $\alpha_\ell b_\ell^* > \alpha_h b_h^*$ . Therefore, any equilibrium must take the form in case (c), with  $b_\ell^* = b_\ell^+$  and  $b_h^* = b_h^-$ .

Now, observe that  $b_\ell^+$  and  $b_h^-$  are both continuous and strictly decreasing in  $r$  as long as  $\alpha_\ell, \alpha_h < 1$ . At the rate  $r = \bar{r}(\alpha_h)$ , we have  $b_h^- = \beta_\ell$  and  $b_\ell^+ > \beta_h$ , so there is excess demand for funds. At the rate  $r = \underline{r}(\alpha_h)$ , we have  $b_\ell^+ = \beta_h$  and  $b_h^- < \beta_\ell$ , so there is excess supply of funds. By continuity of  $b_h^-, b_\ell^+$ , there must exist an  $r^* \in (\underline{r}(\alpha_h), \bar{r}(\alpha_h))$  at which market-clearing is satisfied. Given that  $b_\ell^+ + b_h^-$  is strictly decreasing in  $r$ , there is only one such  $r^*$ . Such an  $r^*$ , with the associated values of  $b_h^-$  and  $b_\ell^+$  at that interest rate, constitutes a unique equilibrium when  $\alpha > \bar{\alpha}$ .

Note that at any such  $r$ , we have  $b_h^* = b_h^- < \beta_\ell$  and  $b_\ell^* = b_\ell^+ > \beta_h$ .

(ii) As shown in part (i), in equilibrium we have  $b_h^* = b_h^- = g(1 + r^*\lambda(1 - \alpha_h) + \tau\alpha_h)$  and  $b_\ell^* = b_\ell^+ = g(1 + r^*\lambda(1 - \alpha_\ell))$ . That is,  $f'(b_h^*) = 1 + r^*\lambda(1 - \alpha_h) + \tau\alpha_h$  and  $f'(b_\ell^*) = 1 + r^*\lambda(1 - \alpha_\ell)$ . Therefore,  $f'(b_h^*) - f'(b_\ell^*) = -r^*\lambda(\alpha_h - \alpha_\ell) + \tau\alpha_h$ .

Now there are two cases to consider:

(a)  $\tau < \bar{\tau}$ . Then, as shown in Proposition 1 of the paper, the second-best outcome sets  $f'(b_h^s) - f'(b_\ell^s) = \tau(\alpha_h + \alpha_\ell)$ . Now, in part (i) we have shown that  $r^* > \underline{r}(\alpha_h) > 0$ . Therefore,  $-r^*\lambda(\alpha_h - \alpha_\ell) + \tau\alpha_h < \tau(\alpha_h + \alpha_\ell)$ .

That is,  $f'(b_h^*) - f'(b_\ell^*) < f'(b_h^s) - f'(b_\ell^s)$ . Now, it follows from market-clearing in the interbank market that  $b_h^* + b_\ell^* = b_h^s + b_\ell^s = 2I$ . Therefore, it must be that  $b_h^* > b_h^s$  and  $b_\ell^* < b_\ell^s$ .

(b)  $\tau \geq \bar{\tau}$ . In this case, as shown in Proposition 1 of the paper, the second-best outcome sets  $b_h^s = \beta_\ell$  and  $b_\ell^s = \beta_h$ , where  $\beta_i = \frac{\alpha_i}{\alpha_i + \alpha_{-i}} 2I$ . Now, when  $r < \underline{r}(\alpha_h)$ , we have  $b_h^* > \beta_\ell$ , and when  $r > \bar{r}(\alpha_h)$ , we have  $b_\ell^* < \beta_h$ . It follows immediately that  $b_h^* > \beta_\ell$  and  $b_\ell^* < \beta_h$ .

(iii) As  $\alpha_h \beta_\ell = \alpha_\ell = \beta_h$ , when  $b_h^* > \beta_\ell$  and  $b_\ell^* < \beta_h$ , it follows that  $\alpha b_h^* > \alpha_\ell b_\ell^*$ , so bank  $h$  makes a net transfer to bank  $\ell$  at date 2.

(iv) Consider  $\alpha_h$  approaching  $\bar{\alpha}$  from above. In the limit as  $\alpha_h \rightarrow \bar{\alpha}$ , we have  $b_h^* = b_h^- = \beta_\ell < b_\ell^+ = b_\ell^* = b_\ell^+ = \beta_h$ . Further,  $b_h^-$  and  $b_\ell^+$  are continuous in  $\alpha_h$ . Thus, for  $\alpha_h$  close to but strictly greater than  $\bar{\alpha}$ , it must continue to be the case that  $b_h^* = b_h^- < b_\ell^* = b_\ell^+$ .

Next, consider  $\alpha_h = 1$ . At this value, we have  $b_h^- = g(1 + \tau)$  and  $b_\ell^+ = g(1 + \lambda r^*(1 - \alpha))$ . Now, when  $\alpha_h > \bar{\alpha}$ , we have  $r^* > \bar{r} = \frac{f'(\beta_h) - 1}{\lambda(1 - \alpha_\ell)}$ , so that  $\lambda r^*(1 - \alpha_\ell) > f'(\beta_h) - 1$ . Further, from Assumption 1 part (i), we have  $\tau < f'(\beta_h) - 1$ . Thus,  $\lambda r^*(1 - \alpha_\ell) > \tau$ , so that when  $\alpha_h = 1$ , we have  $b_h^* = b_h^- > b_\ell^* = b_\ell^+$ .

Now,  $\lambda r^*(1 - \alpha_\ell) > \tau$  implies that  $\lambda r^* > \tau$ . Consider an increase in  $\alpha_h$ . Keeping  $r^*$  fixed, this leads to a decrease in  $g(1 + r^*\lambda(1 - \alpha_h) + \tau\alpha_h)$ , and hence to an increase in  $b_h^*$ . The increase

in  $\alpha_h$  has no effect on  $b_\ell^*$  when  $r^*$  is held fixed. Therefore, there is now excess demand for funds, which means that  $r^*$  must increase. Therefore, in the new equilibrium,  $r^*$  is higher, and hence  $b_\ell^*$  decreases, which means that  $b_h^*$  must increase.

As  $b_h^*$  is increasing in  $\alpha_h$  and  $b_\ell^*$  is decreasing in  $\alpha_h$ , it follows that there exists some threshold value  $\hat{\alpha}$  such that  $b_h^* < b_\ell^*$  when  $\alpha_h < \hat{\alpha}$  and  $b_h^* > b_\ell^*$  when  $\alpha_h > \hat{\alpha}$ .

(v) As in the proof of Proposition 2 part (iv), we have  $z_\ell^* > z_h^*$  if and only if  $(1-2\alpha_\ell)b_\ell^* > (1-2\alpha_h)b_h^*$ . Consider the following two cases:

- (a) Suppose  $\alpha > \bar{\alpha}$  (so that we are in a payments equilibrium)  $f(x) = A \ln x$  where  $A > 0$ . In this case, when  $\tau > 0$  we have  $b_h^* = \frac{A}{1+r^*\lambda(1-\alpha_h)+\tau\alpha_h}$  and  $b_\ell^* = \frac{A}{1+r^*\lambda(1-\alpha_\ell)}$ . Therefore, the condition  $(1-2\alpha_\ell)b_\ell^* > (1-2\alpha_h)b_h^*$  holds if and only if  $\frac{1-2\alpha_\ell}{1+r^*\lambda(1-\alpha_\ell)} > \frac{1-2\alpha_h}{1+r^*\lambda(1-\alpha_h)+\tau\alpha_h}$ . As  $r^* > 0$  given Assumption 1 part (i), it follows (as in the proof of part (i)) that  $\frac{1-2\alpha_\ell}{1+r^*\lambda(1-\alpha_\ell)} > \frac{1-2\alpha_h}{1+r^*\lambda(1-\alpha_h)} > \frac{1-2\alpha_\ell}{1+r^*\lambda(1-\alpha_\ell)} > \frac{1-2\alpha_h}{1+r^*\lambda(1-\alpha_h)+\tau\alpha_h}$ . Thus,  $z_\ell^* > z_h^*$ . Market-clearing now implies that  $z_\ell^* > 0 > z_h^*$ .
- (b) Suppose  $\alpha_h \geq \max\{\frac{1}{2}, \hat{\alpha}\}$ . If  $\alpha_\ell \leq \frac{1}{2}$ , it is immediate that  $(1-2\alpha_\ell)b_\ell^* > (1-2\alpha_h)b_h^*$ . If  $\alpha_\ell > \frac{1}{2}$ , then  $\alpha_h > \alpha_\ell \implies 0 > 1-2\alpha_\ell > 1-2\alpha_h$ . Now, from part (iv),  $\alpha > \hat{\alpha}$  implies that  $b_\ell^* < b_h^*$ , so that  $(1-2\alpha_\ell)b_\ell^* > (1-2\alpha_h)b_h^*$ . Hence, for  $\alpha_h \geq \frac{1}{2}$ , we have  $z_\ell^* > z_h^*$ . Market-clearing now implies that  $z_\ell^* > 0 > z_h^*$ . ■

#### Proof of Proposition 4

Suppose that  $\tau < \bar{\tau}$  and  $\alpha_h > \bar{\alpha}$ .

(i) When  $\tau < \bar{\tau}$  and  $\tau$  decreases by a small amount,  $f'(b_h) - f'(b_\ell) = \tau(\alpha_h + \alpha_\ell)$  strictly decreases. As  $b_h + b_\ell$  is fixed at  $2I$  by market-clearing, it follows that  $b_h$  strictly decreases and  $b_\ell^*$  strictly increases.

(ii) When  $\alpha_h > \bar{\alpha}$ , in a market equilibrium, we have  $b_h^* = b_h^- = g(1 + r^*\lambda(1 - \alpha_h) + \tau\alpha_h)$ , and  $b_\ell^* = b_\ell^+ = g(1 + r^*\lambda(1 - \alpha_\ell))$ .

Suppose that  $\alpha_h > \bar{\alpha}$  and consider equilibrium values  $b_h^*$  and  $b_\ell^*$  at some  $\tau$ . Consider a small decrease in  $\tau$ , keeping  $r^*$  fixed. Then,  $1 + r^*\lambda(1 - \alpha_h) + \tau\alpha_h$  decreases, so that  $b_h^- = g(1 + r^*\lambda(1 - \alpha_h) + \tau\alpha_h)$  increases. With  $r^*$  fixed, the change in  $\tau$  has no direct effect on  $b_\ell^+$ . Therefore, if  $r^*$  is kept fixed, there is an excess demand for investable funds. Hence,  $r^*$  must increase in equilibrium.

The increase in  $r^*$  in turn implies an decrease in  $b_\ell^* = b_\ell^+ = g(1 + r^*\lambda(1 - \alpha_\ell))$ . Market-clearing now implies that  $b_h^* = b_h^-$  increases. ■

#### Proof of Proposition 5

Suppose that  $\tau < \bar{\tau}$  and  $\alpha_h > \bar{\alpha}$ . Consider a small increase in  $\alpha_h$ .

(i) When  $\tau < \bar{\tau}$ , as Proposition 1 shows, in the solution to the second-best problem we have  $f'(b_h^s) - f'(b_\ell^s) = \tau(\alpha_h + \alpha_\ell)$ . Therefore, when  $\alpha_h$  increases by a small amount,  $f'(b_h^s) - f'(b_\ell^s)$  strictly increases. As  $b_h + b_\ell$  is fixed at  $2I$  by market-clearing, it follows that  $b_h$  strictly increases and  $b_\ell^s$  strictly decreases.

(ii) This part was proved in part (iv) of Proposition 3. ■

### Proof of Proposition 6

The proof proceeds in three steps.

*Step 1:* Suppose  $\alpha > \bar{\alpha}$ . Then, for every  $\gamma \in [0, 1]$ , in equilibrium  $b_h^* = b_h^-(\gamma) = g(1 + r^*\lambda(1 - \alpha_h) + \gamma\tau\alpha_h)$  and  $b_\ell^* = b_\ell^+(\gamma) = g(1 + r^*\lambda(1 - \alpha_\ell) - (1 - \gamma)\tau\alpha_\ell)$ .

Given an interbank interest rate, for each  $i = h, \ell$ , re-define  $b_i^+(\gamma) = g(1 + r\lambda(1 - \alpha_i) - (1 - \gamma)\tau\alpha_i)$  and  $b_i^-(\gamma) = g(1 + r\lambda(1 - \alpha_i) + \tau\alpha_i)$ . Next, re-define

$$\underline{r}(\alpha \mid \alpha_\ell, \tau, \gamma, I) = \frac{f'(\beta_\ell) - 1 - \gamma\tau\alpha}{\lambda(1 - \alpha)} \quad (27)$$

$$\bar{r}(\alpha \mid \alpha_\ell, \tau, \gamma, I) = \frac{f'(\beta_h) - 1 - (1 - \gamma)\tau\alpha_\ell}{\lambda(1 - \alpha_\ell)} \quad (28)$$

The arguments in Lemma 3 now follow, and imply that if  $\underline{r}(\alpha_h) \leq \bar{r}(\alpha_h)$ , there is a no-payments equilibrium. Similarly, the arguments in part (i) of Proposition 3 imply that if  $\underline{r}(\alpha_h) > \bar{r}(\alpha_h)$ , there is a payments equilibrium in which bank  $h$  is a net payer at date 2. Now, consider any value  $\hat{\gamma} \in (0, 1]$ , and find  $\alpha_h$  such that  $\underline{r}(\alpha_h) = \bar{r}(\alpha_h)$ . If  $\gamma$  is decreased slightly,  $\underline{r}$  increases, whereas  $\bar{r}$  decreases. Thus, if  $\underline{r}(\alpha_h \mid \hat{\gamma}) \geq \bar{r}(\alpha_h \mid \hat{\gamma})$ , then  $\underline{r}(\alpha_h \mid \gamma) > \bar{r}(\alpha_h \mid \gamma)$  for all  $\gamma < \hat{\gamma}$ . Proposition 1 has shown that when  $\alpha_h > \bar{\alpha}$ , we have  $\underline{r}(\alpha_h \mid \gamma = 1) > \bar{r}(\alpha_h \mid \gamma = 1)$ . Thus, the inequality is preserved for all  $\gamma < 1$ , and hence for all values  $\gamma \in [0, 1]$ , there is a payments equilibrium.

The arguments in the remainder of the proof of Proposition 3 now follow. In equilibrium, bank  $h$  is a net payer at time 2, and bank  $\ell$  is a net recipient. Then, it follows that the profit function of bank  $h$  at a given interest rate  $r$  is  $\pi_h = f(b_h) - b_h - rz_h - \tau(\alpha_h b_h - \alpha_\ell b_\ell)$ , from which it follows that in equilibrium  $b_h^* = b_h^-(\gamma) = g(1 + r^*\lambda(1 - \alpha_h) + \gamma\tau\alpha_h)$ . Similarly, the profit function of bank  $\ell$  is  $\pi_\ell = f(b_\ell) - b_\ell - rz_\ell - (1 - \tau)(\alpha_h b_h - \alpha_\ell b_\ell)$ , from which it follows that in equilibrium  $b_\ell^* = b_\ell^+(\gamma) = g(1 + r^*\lambda(1 - \alpha_\ell) - (1 - \gamma)\tau\alpha_\ell)$ .

*Step 2:*  $\frac{db_h^*}{d\gamma} < 0$  and  $\frac{db_\ell^*}{d\gamma} > 0$ .

The market-clearing constraint implies that  $b_h^* + b_\ell^* = 2I$ . Totally differentiating with respect

to  $\gamma$ , we have  $\frac{db_h^*}{d\gamma} + \frac{db_\ell^*}{d\gamma} = 0$ . That is,

$$g'(1 + r^* \lambda(1 - \alpha_h) + \gamma \tau \alpha_h) \left\{ \lambda(1 - \alpha_h) \frac{dr^*}{d\gamma} + \tau \alpha_h \right\} + g'(1 + r^* \lambda(1 - \alpha_\ell) - (1 - \gamma) \tau \alpha_\ell) \left\{ \lambda(1 - \alpha_\ell) \frac{dr^*}{d\gamma} + \tau \alpha_\ell \right\} = 0. \quad (29)$$

As  $g'(\cdot) < 0$ , if  $\frac{dr^*}{d\gamma} \geq 0$ , the left-hand side of the last equation is strictly negative, and cannot equal zero. Therefore, it must be that  $\frac{dr^*}{d\gamma} < 0$ .

Now, observe that when  $\frac{dr^*}{d\gamma} < 0$ , we have  $\lambda(1 - \alpha_h) \frac{dr^*}{d\gamma} + \tau \alpha_h > \lambda(1 - \alpha_\ell) \frac{dr^*}{d\gamma} + \tau \alpha_\ell$ . As the two terms  $\lambda(1 - \alpha_h) \frac{dr^*}{d\gamma} + \tau \alpha_h$  and  $\lambda(1 - \alpha_\ell) \frac{dr^*}{d\gamma} + \tau \alpha_\ell$  must have opposite signs to satisfy equation (29), it must be that  $\lambda(1 - \alpha_h) \frac{dr^*}{d\gamma} + \tau \alpha_h > 0 > \lambda(1 - \alpha_\ell) \frac{dr^*}{d\gamma} + \tau \alpha_\ell$ . It now follows that  $\frac{db_h^*}{d\gamma} = g'(1 + r^* \lambda(1 - \alpha_h) + \gamma \tau \alpha_h) \left\{ \lambda(1 - \alpha_h) \frac{dr^*}{d\gamma} + \tau \alpha_h \right\} > 0$ , and similarly  $\frac{db_\ell^*}{d\gamma} < 0$ .

*Step 3:* It is optimal to set  $\gamma^* = 1$ .

From Proposition 3, when  $\gamma = 1$ , we already have  $b_h^* > b_h^s$  (where  $b_h^s$  is the quantity of claims issued by bank  $h$  in the second-best planning outcome) and  $b_\ell^* < b_\ell^s$ . From Step 2, it follows that decreasing  $\gamma$  leads to a further increase in  $b_h^*$  and a further reduction in  $b_\ell^*$ . That is, decreasing  $\gamma$  leads to an equilibrium outcome that is further from the second-best solution, and so reduces the value of the planner's objective function. ■

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